Computational Social Choice: Spring 2009

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Mechanism Design

Mechanism design is concerned with the *design of mechanisms* for collective decision making that favour particular outcomes despite the fact that individuals are pursuing their own interests.

Mechanism design is sometimes referred to as *reverse game theory*. While game theory analyses the strategic behaviour of rational agents in a given game, mechanism design uses these insights to design games inducing certain strategies (and hence outcomes).

We are going to concentrate on mechanism design questions in the context of (private value) *combinatorial auctions*.

Plan for Today

- Revelation Principle: (more or less) formal justification for concentrating on direct-revelation mechanisms
- Review of the Vickrey auction
- Generalisation to combinatorial auctions: VCG mechanism
- Further generalisation to general mechanisms for collective decision making
- Properties: strategy-proofness, efficiency, budget balance
- Problems of the VCG mechanism

Revelation Principle

Claim: Any outcome that can be implemented via an indirect mechanism with dominant strategies can also be implemented by means of a *direct mechanism* (where agents simply reveal their preferences) that makes *truth-telling* a *dominant strategy*.

Intuition: Whatever the agents are doing in the indirect mechanism to transform their true preferences into a strategy, we can use as a "filter" in the corresponding direct mechanism. So, first apply this filter to whatever the agents are reporting and then simulate the indirect mechanism with the filtered input. The outcome will be the same as the outcome we'd get with the indirect mechanism *iff* the agents report their true preferences. ✓

Consequence: can focus on one-step mechanisms

Example: the (direct) Vickrey auction may be regarded as a direct implementation of the (indirect) English auction

Quasi-linear Utilities

- Each agent i has a *valuation function* v_i mapping agreements x (e.g., allocations) to the reals. This could be *any* such function.
- The *utility* u_i of agent i is a function of its valuation $v_i(x)$ for agreement x and the *price* p the agent will have to pay in case x is chosen. In principle, this could be any such function.
- However, we make the (common) assumption that utility functions are *quasi-linear:*

$$u_i(x, p) \quad = \quad v_i(x) - p$$

  That is, utility is linear in both valuation and money.

Reminder: Vickrey Auction

- Motivation: no dominant strategy for the first-price sealed-bid auction, inviting counterspeculation
- Protocol: one round; sealed bid; highest bid wins, but the winner pays the price of the *second highest* bid
- Dominant strategy: bid your true valuation
  - if you bid more, you risk paying too much
  - if you bid less, you lower your chances of winning while still having to pay the same price in case you do win
- How can we generalise this idea to *combinatorial auctions*?

Reinterpreting the Vickrey Pricing Rule

- Distinguish *allocation rule* and *pricing rule*
- Allocation rule: highest bid wins
- Pricing rule: winner pays price offered, but gets a *discount*
- The amount of the discount granted reflects the *contribution* to overall value made by the winner. How can we compute this?
  - Without the winner's bid, the second highest bid would have won. So the contribution of the winner is equal to the difference between the winning and the second highest bid.
  - Subtracting this contribution from the winning bid yields the second highest bid (the Vickrey price).

Vickrey-Clarke-Groves Mechanism

This idea is used in the so-called Vickrey-Clarke-Groves mechanism, which we will introduce next.

We will concentrate on the variant introduced by Edward H. Clarke (for combinatorial auctions), but also mention the more general form of the mechanism as put forward by Theodore Groves.

W. Vickrey. Counterspeculation, Auctions, and Competitive Sealed Tenders. *Journal of Finance*, 16(1):8–37, 1961.

E.H. Clarke. Multipart Pricing of Public Goods. *Public Choice*, 11(1):17–33, 1971.

T. Groves. Incentives in Teams. *Econometrica*, 41(4):617–631, 1973.

Notation

- Set of bidders: $\mathcal{N} = \{1, \ldots, n\}$
- Set of possible agreements (allocations): $\mathcal{X}$
- (True) valuation function of bidder $i \in \mathcal{N}$: $v_i : \mathcal{X} \to \mathbb{R}$
- Valuation function *reported* by bidder $i \in \mathcal{N}$: $\hat{v}_i : \mathcal{X} \to \mathbb{R}$
- Top allocation as chosen by the auctioneer:

$$x^* \in \mathrm{argmax}_{x \in \mathcal{X}} \sum_{j=1}^{n} \hat{v}_j(x)$$

- Allocation that would be chosen if agent i were not to bid:

$$x^*_{-i} \in \mathrm{argmax}_{x \in \mathcal{X}} \sum_{j \neq i} \hat{v}_j(x)$$

VCG Mechanism for Combinatorial Auctions

- *Allocation rule:* solve the WDP and allocate goods accordingly
- *Pricing rule:* Again, the idea is to give each winner a discount reflecting its contribution to overall value. In short, bidder i should pay the following amount:

$$bid_i - (max\text{-}value - max\text{-}value_{-i})$$

The same more formally:

$$\begin{aligned} p_i &= \hat{v}_i(x^*) - \left(\sum_{j=1}^{n} \hat{v}_j(x^*) - \sum_{j \neq i} \hat{v}_j(x^*_{-i}) \right) \\ &= \sum_{j \neq i} \hat{v}_j(x^*_{-i}) - \sum_{j \neq i} \hat{v}_j(x^*) \end{aligned}$$

Alternative interpretation: i pays the sum of the losses in valuation suffered by the other bidders due to i's participation.

Strategy-Proofness

Theorem 1 *In the VCG mechanism, reporting their true valuation is a dominant strategy for each bidder.*

Proof: Consider the situation of bidder i.
Let $h_i = \sum_{j \neq i} \hat{v}_j(x^*_{-i})$. Note that i cannot affect h_i.
We have $p_i = h_i - \sum_{j \neq i} \hat{v}_j(x^*)$ and the utility of i is $v_i(x^*) - p_i$.
Hence, i should try to maximise $v_i(x^*) + \sum_{j \neq i} \hat{v}_j(x^*)$.
But the auctioneer is maximising
$\sum_{j=1}^{n} \hat{v}(x^*) = \hat{v}_i(x^*) + \sum_{j \neq i} \hat{v}_j(x^*)$.
Hence, i can do no better than reporting $\hat{v}_i = v_i$. ✓

Remark: Contrast this with the Gibbard-Satterthwaite Theorem, which (roughly) says that in the context of voting there is no such strategy-proof mechanism. The crucial difference is that now we use money to affect people's incentives and we make specific assumptions regarding the structure of preferences (quasi-linearity).

Generalisation

Our proof suggests a generalisation of the mechanism in a way that preserves strategy-proofness.

Let h_i be *any* function mapping the profile of reported valuations of all bidders except i to the reals (crucially, h_i does *not* depend on $\hat{v}_i$). Then consider the following modified pricing rule:

$$p_i \quad = \quad h_i - \sum_{j \neq i} \hat{v}_j(x^*)$$

The resulting mechanism also makes truth-telling the dominant strategy (same proof).

The specific choice $h_i = \sum_{j \neq i} \hat{v}_j(x^*_{-i})$ is called the *Clarke tax*.
For the remainder of today, suppose the VCG mechanism is defined using the Clarke tax (i.e., we won't be using this generalisation).

A Word on Terminology

Unfortunately, the literature is not that consistent when discussing the many variants of the VCG mechanism. Terms in use include:

- Vickrey Auction; Generalised Vickrey Auction; Clarke Tax and Mechanism; Groves Mechanism; VCG Auction or Mechanism ...

To be precise, we need to fix the following parameters:

- Type of mechanism: single-item auction; multi-unit single-item auction; single-unit combinatorial auction; multi-unit combinatorial auction; mechanism to make a collective decision
- Pricing rule: VCG in its most general form; VCG with the Clarke tax (i.e., h_i = maximum overall value without bidder i)

Unless specified otherwise, we use the following two terms:

- Vickrey Auction: single-item second-price sealed-bid auction
- VCG Mechanism: single-unit combinatorial auction with Clarke tax

Efficiency

By construction, if all bidders submit true valuations (dominant strategy), then the outcome maximises *utilitarian social welfare:*

- payments (including the auctioneer's) sum up to 0; and
- the sum of valuations is being maximised.

But note that this does *not* mean that *revenue* gets maximised as well (unlike for the basic CA without special pricing rules).

Budget Balance

If a mechanism uses monetary side-payments to implement an outcome, the following two properties are of interest:

- *Budget balance:* the sum of all payments is 0
- *Weak budget balance:* the sum is greater than or equal to 0

If we have (weak) budget balance, then the mechanism does not need to get *subsidised*.

For CAs, if we consider both bidders *and the auctioneer*, then (obviously) the sum of payments is always 0 (not the point here).

▶ What about budget balance with respect to bidders alone?

Note that (full) budget balance is actually unattractive for auctions (zero revenue), while weak budget balance is an absolute must.

Weak Budget Balance

If the Clarke tax is used to determine payments, then weak budget balance can usually be guaranteed; in particular:

Theorem 2 *In the context of CAs with free disposal, the VCG mechanism using the Clarke tax is weakly budget balanced.*

Free disposal: the auctioneer can keep goods
(alternatively: agents have monotonic valuation functions)

Proof: Weakly budget balanced *if* sum of payments non-negative *if* each payment non-negative. Hence, weakly budget balanced *if*

$$\begin{array}{rcll} bid_i - (max\text{-}value - max\text{-}value_{-i}) & \geq & 0 & \text{or:} \\ max\text{-}value_{-i} & \geq & max\text{-}value - bid_i & \end{array}$$

This is true: due to free disposal we could throw away the goods given to i in the top allocation; so the other agents can generate at least as much value without i as they do in the top allocation. ✓

Example

The following example shows that weak budget balance cannot be guaranteed if we drop the free disposal assumption:

Agent 1: accept one of $(\{a\}, 90)$, $(\{b\}, 10)$, $(\{a, b\}, 10)$

Agent 2: accept one of $(\{a\}, 20)$, $(\{b\}, 30)$, $(\{a, b\}, 50)$

We end up with the following payments:

Agent 1: $90 - (120 - 50) = +20$

Agent 2: $30 - (120 - 10) = -80$

That is, agent 2 should *receive* money from the auctioneer!

Problems with the VCG Mechanism

Despite their nice game-theoretical properties, CAs using the Clarke tax to determine payments have several problems:

- Low (and possibly even zero) *revenue* for the auctioneer
- *Non-monotonicity:* "better" bids don't entail higher revenue
- *Collusion* amongst (losing) bidders
- *False-name bidding:* bidders may benefit from submitting bids using multiple identities

The following examples illustrating these problems are adapted from Asubel and Milgrom (2006).

L.M. Asubel and P. Milgrom. The Lovely but Lonely Vickrey Auction. In P. Cramton *et al.* (eds.), *Combinatorial Auction*, MIT Press, 2006.

Zero Revenue

There are cases where the VCG mechanism gives zero revenue:

Agent 1: $(\{a\}, 0)$, $(\{b\}, 0)$, $(\{a, b\}, 2)$

Agent 2: $(\{a\}, 2)$, $(\{b\}, 0)$, $(\{a, b\}, 0)$

Agent 3: $(\{a\}, 0)$, $(\{b\}, 2)$, $(\{a, b\}, 0)$

Payments are computed as follows:

Agent 1: 0

Agent 2: $2 - (4 - 2) = 0$

Agent 3: $2 - (4 - 2) = 0$

Note that this problem is independent from whether or not we admit free disposal.

Non-monotonicity

Revenue is not necessarily monotonic in the set of bids or the amounts that are being bid. Consider again the following example:

Agent 1: $(\{a\}, 0)$, $(\{b\}, 0)$, $(\{a, b\}, 2)$

Agent 2: $(\{a\}, 2)$, $(\{b\}, 0)$, $(\{a, b\}, 0)$

Agent 3: $(\{a\}, 0)$, $(\{b\}, 2)$, $(\{a, b\}, 0)$

As seen before, revenue for this example is 0.

If we either *remove* agent 3 or *decrease* the amount agent 3 is offering for item b, then revenue will *increase*.

Collusion

The VCG mechanism is not collusion-proof: if bidders work together they can manipulate the mechanism. Consider the following example:

Agent 1: $(\{a\}, 0)$, $(\{b\}, 0)$, $(\{a, b\}, 4)$

Agent 2: $(\{a\}, 1)$, $(\{b\}, 0)$, $(\{a, b\}, 0)$

Agent 3: $(\{a\}, 0)$, $(\{b\}, 1)$, $(\{a, b\}, 0)$

Agent 1 wins and pays $4 - (4 - 2) = 2$.

But if the two losing bidders collude and increase their two bids to $(\{a\}, 4)$ and $(\{b\}, 4)$, respectively, they can obtain the items for free.

False-name Bidding

False-name bidding (aka. *shill* or *pseudonymous* bidding) is yet another form of manipulation the VCG mechanism is exposed to. Example:

Agent 1: $(\{a\},0)$, $(\{b\},0)$, $(\{a,b\},4)$

Agent 2: $(\{a\},1)$, $(\{b\},1)$, $(\{a,b\},2)$

Agent 1 wins. But agent 2 could instead submit bids *under two names:*

Agent 1: $(\{a\},0)$, $(\{b\},0)$, $(\{a,b\},4)$

Agent 2: $(\{a\},4)$, $(\{b\},0)$, $(\{a,b\},0)$

Agent 2': $(\{a\},0)$, $(\{b\},4)$, $(\{a,b\},0)$

Agent(s) 2 (and 2') will win and not pay anything! This form of manipulation is particularly critical for *electronic* auctions, as it is easier to create multiple identities online than it is in real life.

M. Yokoo. Pseudonymous Bidding in Combinatorial Auctions. In P. Cramton *et al.* (eds.), *Combinatorial Auction*, MIT Press, 2006.

Computational Issues

- Observe that computing the Clarke tax requires solving an additional n winner determination problems.
- That means, the auctioneer has to solve $n + 1$ NP-hard optimisation problems.
- If allocations and prices are not being computed according to the *optimal* solutions to these problems, then we cannot guarantee strategy-proofness anymore.

Summary

We have introduced the Vickrey-Clarke-Groves mechanism, a mechanism for collective decision making that makes truth-telling the dominant strategy.

- Distinguish most general form of the VCG mechanism and the variant where the Clarke tax is used to determine payments
- Additional properties: efficiency and weak budget balance (the latter under suitable conditions)
- Drawbacks: high complexity, potential for low revenue, manipulation through collusion or use of false-name bids, ...
- Restriction: applies to agents with quasi-linear utilities only

References

The first chapter of the *Combinatorial Auctions* book defines the VCG mechanism for CAs (the variant with Clarke tax) and gives a proof sketch for the strategy-proofness result:

- L.M. Asubel and P. Milgrom. The Lovely but Lonely Vickrey Auction. In P. Cramton *et al.* (eds.), *Combinatorial Auction*, MIT Press, 2006.

Another possible starting point is the following paper:

- H.R. Varian. *Economic Mechanism Design for Computerized Agents*. Proc. Usenix Workshop on Electronic Commerce, 1995.