Homework #2

Deadline: Thursday, 20 April 2017, 19:00

Question 1 (10 marks)

Recall that under the *antiplurality rule*, also known as the *veto rule*, the voters rank the alternatives, and the alternative(s) ranked last the least often win(s). Thus, this is the positional scoring rule with scoring vector $(1, \ldots, 1, 0)$. The purpose of this exercise is to find a number of different characterisations of this rule.

(a) Find a consensus criterion such that the antiplurality rule is characterised by that criterion and the *discrete distance*.

(b) Find a way of measuring distances such that the antiplurality rule is characterised by the *unanimous winner* consensus criterion and that distance.

(c) Find a noise model such that the corresponding maximum likelihood estimator is equivalent to the antiplurality rule.

Question 2 (10 marks)

Recall that, when you design a voting rule, it is not always possible to achieve resoluteness if we also require anonymity and neutrality. For example, it is easy to see that this combinations of desiderata is impossible to satisfy when there are two alternatives and two voters.

For this exercise, we focus on elections with *three alternatives* and *n voters*. Suppose we accept that resoluteness is hard to achieve, but that we at least want to have a voting rule that *never returns a three-way tie* between all three alternatives, besides being *anonymous* and *neutral*. For some values of n this is possible, while for others it is impossible. Provide a full characterisation of when it is possible and when it is impossible. For the cases for which it is possible, define a voting rule that has all the desired properties. For the cases for which it is impossible, prove that this really is so.

Question 3 (10 marks)

How many different (possibly irresolute) voting rules are there for scenarios with *three alternatives* and *three voters*? How many of them are anonymous? How many are neutral? How many are both anonymous and neutral? How many are resolute? How many are resolute and anonymous? How many are resolute and neutral? How many are resolute, anonymous, and neutral? Justify your answers.

Question 4 (10 marks)

The purpose of this exercise is to investigate what happens to Arrow's Theorem, in its formulation for resolute social choice functions discussed in class, if we replace the Pareto Principle by the seemingly more basic surjectivity condition. Recall that we had defined surjectivity in the context of our discussion of the Muller-Satterthwaite Theorem.

(a) Show that the Pareto Principle is strictly stronger than surjectivity. That is, show that every Paretian resolute social choice function is surjective and that there exists a surjective resolute social choice function that is not Paretian.

(b) Show that Arrow's Theorem ceases to hold when we replace the Pareto Principle by surjectivity. That is, show that there exists a resolute social choice function that is surjective, independent, and nondictatorial.