Participatory Budgeting

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Plan of the Day

Today we investigate *Participatory Budgeting* (PB), a loosely defined range of democratic tools used to involve citizens in budgeting decisions.

We shift a bit from the purely theoretical approach that we took for most of the course, and move closer to real-world applications of computational social choice.

Throughout the lecture, we will:

- Introduce the standard approval-based model for PB;
- Discuss how to embed it in Judgment Aggregation (JA), somewhat efficiently;
- Develop the usual axiomatic analysis, with a specific focus on fairness requirements.

Aziz and Shah "Participatory Budgeting: Models and Approaches" (2020)
Cabannes "Participatory Budgeting: A significant Contribution to Participatory Democracy" (2004)
Shah "Participatory Budgeting" (2007)

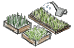
1. Introduction

© Marianne de Heer Kloots

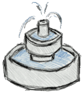
1000€

2000€

2500€



2500€

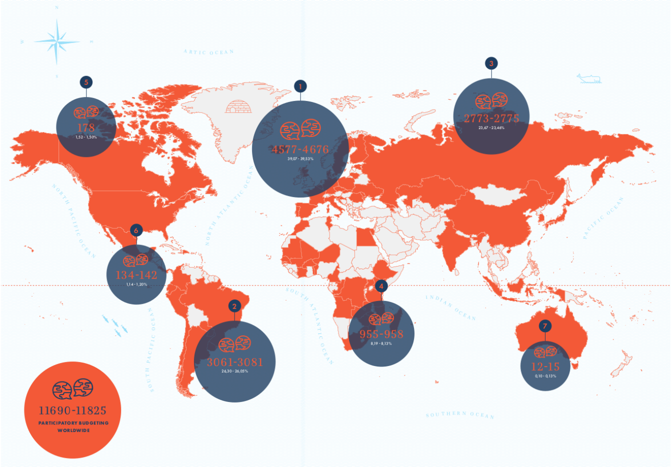


5000€

 : 7000€

Aggregating preferences regarding costly alternatives to find an outcome satisfying the budget constraint.

From Brazil in the 1980s...



Dias, Enríquez, and Júlio "The Participatory Budgeting World Atlas" (2019)

Budget gekozen: € 86.865 | Budget over: € 3.745 | Omhoog

MoTown, Groen voor en door de buurt

Meer groen in de buurt

› Lees meer

€ 9.465 | Toegevoegd

Groene verbinding tussen de Transv...

Meer groen in de buurt

› Lees meer

€ 23.700 | Toegevoegd

Een minibos en containertuintje bij ...

Meer groen in de buurt

› Lees meer

€ 7.500 | Toegevoegd

Moestuinen in het Dapperhuis

Meer groen in de buurt

› Lees meer

€ 23.000 | Toegevoegd

Plantenbakken verbeteren op de Ee...

Meer groen in de buurt

› Lees meer

€ 17.000 | Toegevoegd

Vergroenen van de veranda van het ...

Meer groen in de buurt

› Lees meer

€ 10.000 | Geen budget

https://oostbegroot.amsterdam.nl/oudoost

Approval-Based Participatory Budgeting

A PB *instance* is a tuple $I = \langle \mathcal{P}, c, b \rangle$. $\mathcal{P} = \{p_1, \ldots, p_m\}$ is the set of *projects*. Each project $p \in \mathcal{P}$ has a *cost* $c(p) \in \mathbb{N}$. For $P \subseteq \mathcal{P}$, we define $c(P) = \sum_{p \in P} c(p)$. The *budget limit* is $b \in \mathbb{N}$.

The outcome of a PB instance is a *budget allocation* $\pi \subseteq \mathcal{P}$ such that $c(\pi) \leq b$. The set of all budget allocations for an instance I is $\mathcal{A}(I) = \{\pi \subseteq \mathcal{P} \mid c(\pi) \leq b\}$.

To select a suitable budget allocation, we ask a set of *agents* $N = \{1, \ldots, n\}$ to submit their *preferences* regarding the projects. We will focus on *approval ballots*, i.e., each agent submits a ballot $A_i \subseteq \mathcal{P}$ indicating the projects they approve of. The vector $\mathbf{A} = (A_1, \ldots, A_n)$ is a *profile*.

Other ballots could be—and have been—considered:

- k-approval ballots: Approval ballots of size no more than k;
- knapsack ballots: Approval ballots costing no more than b;
- ranking of the projects by preference;
- ranking of the projects by preference per unit of money…

Greedy Approval Mechanism

A resolute PB *mechanism* F is a function taking as input an instance I and a profile $\mathbf{A}$, and returning a budget allocation $F(I, \mathbf{A}) \in \mathcal{A}(I)$.

The most widely used mechanism in practice is *greedy approval* that proceeds as follows:

Definition: Greedy Approval Mechanism

First, order the projects based on the number of agents approving of it, breaking ties lexicographically (based on the name of the projects).

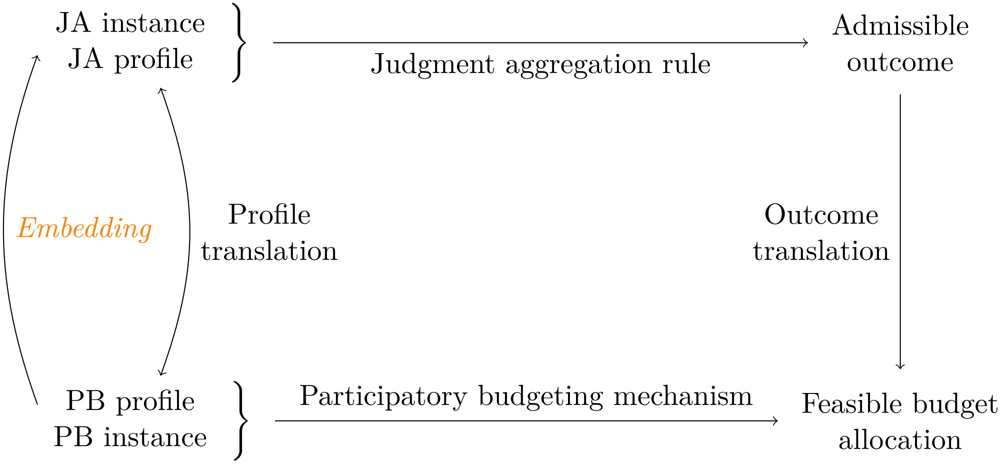
Then, accept the projects in this order, skipping the ones rendering the outcome infeasible.

First observation: Greedy approval always return a feasible subset of projects.

Second observation: The outcome is also *exhaustive*: none of non-selected projects could be selected without violating the budget constraint.

2. Embedding Into Judgment Aggregation

The focus of the course has been on judgment aggregation (JA) so far, can we do PB with JA?



A Naive Approach

Let us embed a PB instance $I = \langle \mathcal{P}, c, b \rangle$ into an instance of *binary aggregation with integrity constraints* with the agenda $\Phi = \{x_p \mid p \in \mathcal{P}\} \cup \{\neg x_p \mid p \in \mathcal{P}\}$.

An approval ballots $A_i \subseteq \mathcal{P}$ is translated into $J_i = \{x_p \mid p \in A_i\} \cup \{\neg x_p \mid p \in \mathcal{P} \setminus A_i\}$. A JA outcome (i.e. a judgment) can be translated similarly into a subset of projects.

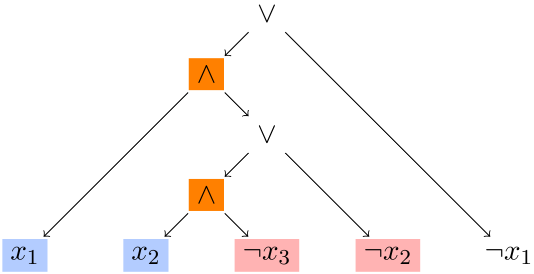
The integrity constraint should reflect the budget constraint. *Discussion:* How to do so?

A naive approach:

$$\Gamma = \bigvee_{\pi \in \mathcal{A}(I)} \left(\bigwedge_{p \in \pi} x_p \wedge \bigwedge_{p \in \mathcal{P} \setminus \pi} \neg x_p \right).$$

Discussion: Why is this naive? What is the issue here? If we do better, are we still good?

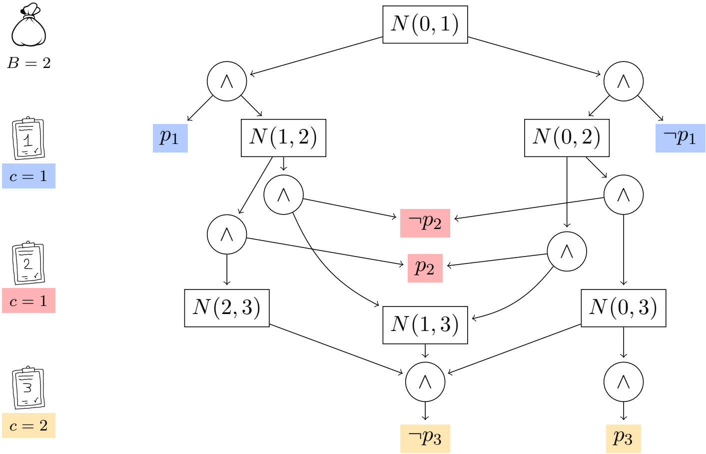
One way to escape the intractability of JA rules is to aim for an *island of tractability*. For instance, the outcome of the Kemeny rule is polynomial-time computable when Γ is a *DNNF circuits*.



De Haan "Hunting for Tractable Languages for Judgment Aggregation" (2018)

Embedding Into DNNF Circuits

Denote by $N(i,j)$ the $\vee$-node at which amount i has been spent and p_j is under considerations.



Discussion: What is the size of the embedding?

Who Cares About Embeddings?

Embedding into JA allows to use the expressivity of JA to account for many small variations.

We can easily extend the embedding for the following cases:

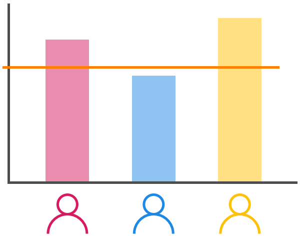
- *Multiple resources*: the cost of the projects is multidimensional;
- *Dependencies between projects*: whether some projects can be selected depends on the status of, potentially several, other projects;
- *Quotas over categories of projects*: projects are gathered in some categories and there are upper and lower bounds on what can be accepted from each category.

In addition, any axiomatic result proved on the JA side that is independent on the integrity constraint will hold in all of these variations.

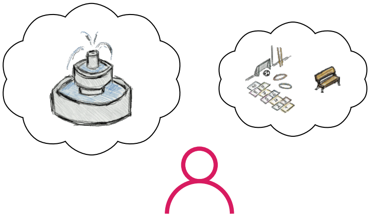
Rey, Endriss, and Haan "Designing Participatory Budgeting Mechanisms Grounded in Judgment Aggregation" (2020)

Let us leave JA on the side for now and discuss PB in more depth. Three main topics have been studied in the literature:

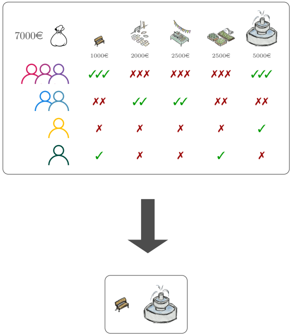
Fairness Requirements



Incentive Compatibility



Algorithmic Efficiency



3. Fairness in Participatory Budgeting

What is Fairness?

In the context of PB, the literature mainly focuses on *Distributive Justice*: the just allocation of social resources to the members of society (see plato.stanford.edu/entries/justice-distributive).

Most of the existing concepts are *welfare-based*: the fairness of an allocation is measured through its effect on agents' satisfaction. We thus need to define ways to measure satisfaction.

Discussion: How to measure satisfaction in approval-based PB?

Two main proxies are used:

- *Cardinality-satisfaction*: $sat_i^{card}(\pi) = |A_i \cap \pi|$
- *Cost-satisfaction*: $sat_i^{cost}(\pi) = c(A_i \cap \pi)$

Other ideas: getting full satisfaction functions, equity of resources instead of sat-based fairness...

Peters, Pierczynski, and Skowron "Proportional Participatory Budgeting with Additive Utilities" (2021)
Maly, Rey, Endriss, and Lackner "Effort-Based Fairness for Participatory Budgeting" (2022)

The Core of Participatory Budgeting

Definition: Core

A budget allocation π is in the *core* of PB if there exists no coalition $C \subseteq N$ for which there is a subset of projects $P \subseteq \mathcal{P}$ such that:

$$c(P) \leq b \cdot \frac{|C|}{n},$$

and such that for all agents $i \in C$, $sat_i^{cost}(P) > sat_i^{cost}(\pi)$.

Interpretation: An allocation is in the core, if no group of agents C could use the share of budget they deserve $b \cdot \frac{|C|}{n}$ to buy some subset of projects P that would make them all more satisfied.

Observation: This can be defined for any satisfaction function, and not just for sat^{cost}.

Open Problem: Can we always find a budget allocation in the core?

Restricting Possible Deviations — EJR

We do not know if we can guarantee the selected budget allocation to be in the core. We get more positive results if we relax a bit the definition of the core.

Definition: Extended Justified Representation (EJR)

A budget allocation π satisfies *EJR* if there exists no coalition $C \subseteq N$ for which there is a subset of projects $P \subseteq \bigcap_{i \in C} A_i$ (C is called P-cohesive) such that:

$$c(P) \leq b \cdot \frac{|C|}{n},$$

and such that for all agents $i \in C$, $sat_i^{cost}(P) > sat_i^{cost}(\pi)$.

Peters, Pierczynski, and Skowron "Proportional Participatory Budgeting with Additive Utilities" (2021)

The Greedy Cohesive Mechanism — Is feasible

Definition: Greedy Cohesive Mechanism

Initialise π and $N^\star$ to the empty set. While there are non-empty $C \subseteq N \setminus N^\star$ and $P \subseteq \mathcal{P} \setminus \pi$ such that C is P-cohesive, go through the round, call it j:

- Choose $C_j \subseteq N \setminus N^\star$ and $P_j \subseteq \mathcal{P} \setminus \pi$ such that C_j is P_j-cohesive and maximizes $c(P_j)$
- Select all the projects in P_j: $\pi \leftarrow \pi \cup P_j$
- Remove agents in C_j from considerations: $N^\star \leftarrow N^\star \cup C_j$

Claim: The mechanism always terminates and returns a feasible outcome.

$$c(\pi) \;=\; \sum_j c(P_j) \;\leq\; \sum_j b \cdot \frac{|C_j|}{n} \;=\; \frac{b}{n} \cdot \sum_j |C_j| \;\leq\; b.$$

- $(P_1, P_2, \ldots)$ is a partition of π;
- All C_j are P_j-cohesive;
- All $C_1, C_2, \ldots$ are disjoint.

The Greedy Cohesive Mechanism — Satisfies EJR

Definition: Greedy Cohesive Mechanism

Initialise π and $N^\star$ to the empty set. While there are non-empty $C \subseteq N \setminus N^\star$ and $P \subseteq \mathcal{P} \setminus \pi$ such that N is P-cohesive, go through the round, call it j:

- Choose $C_j \subseteq N \setminus N^\star$ and $P_j \subseteq \mathcal{P} \setminus \pi$ such that C_j is P_j-cohesive and maximizes $c(P_j)$

Claim: The outcome satisfies EJR. Assume not, i.e., there exist C, a P-cohesive group such that for all $i \in C$, $c(A_i \cap P) > c(A_i \cap \pi)$. Note that this entails $P \not\subseteq \pi$.

Assume that $C \cap N^\star \neq \emptyset$, and let j be the smallest round such that there is $i^\star \in C \cap C_j$. C and P were not chosen in this round, so $c(P_j) \geq c(P)$. Since $i^\star$ was moved to $N^\star$ at this round, $P_j \subseteq A_{i^\star}$. We thus have $c(A_{i^\star} \cap \pi) \geq c(A_{i^\star} \cap P_j) \geq c(A_{i^\star} \cap P)$. π thus satisfies EJR for C.

Assume that $C \cap N^\star = \emptyset$. As the mechanism always terminates, this implies $P \cap \pi \neq \emptyset$. Then, run the same proof for C that is $(P \setminus \pi)$-cohesive. Iterating this would yield to a contradiction, either in the above, or here showing that $P \subseteq \pi$.

Definition: Greedy Cohesive Mechanism

Initialise π and $N^\star$ to the empty set. While there are non-empty $C \subseteq N \setminus N^\star$ and $P \subseteq \mathcal{P} \setminus \pi$ such that C is P-cohesive, go through the round, call it j:

- Choose $C_j \subseteq N \setminus N^\star$ and $P_j \subseteq \mathcal{P} \setminus \pi$ such that C_j is P_j-cohesive and maximizes $c(P_j)$
- Select all the projects in P_j: $\pi \leftarrow \pi \cup P_j$
- Remove agents in C_j from considerations: $N^\star \leftarrow N^\star \cup C_j$

Discussion: What is the running time here? Can we do better?

EJR Budget Allocations Cannot be Computed in Polynomial-Time

We want to show that one cannot compute a budget allocation satisfying EJR in polynomial-time.

Question: How to make this formal?

Claim: There exists no poly-time algorithm $\mathfrak{A}$ returning an EJR budget allocation unless $\mathtt{P} = \mathtt{NP}$.

Subset Sum

Input: A set of integers $S = \{s_1, \ldots, s_m\}$ and a target $t \in \mathbb{N}$.
Question: Is there an $X \subseteq S$ such that $\sum_{x \in X} x = t$?

PB instance I: $\mathcal{P} = \{p_1, \ldots, p_m\}$ with cost $c(p_j) = s_j$ and $b = t$. A single agent approves of $\mathcal{P}$.

In this instance, if there exists $\pi^\star \in \mathcal{A}(I)$ such that $c(\pi^\star) = b$, then every π satisfying EJR should be such that $c(\pi) = b$, since the unique voter would be $\pi^\star$-cohesive. This is the case if and only if (S, t) is a positive instance of Subset Sum.

We can thus: transform the Subset Sum into the PB in polynomial-time; use $\mathfrak{A}$ to compute in polynomial-time π that satisfies EJR; answer the Subset Sum problem with “Yes” if $c(\pi) = b$ and “No” otherwise. That would imply $\mathtt{P} = \mathtt{NP}$.

Getting Polynomial-Time Computation

We have made some progress: the core may not be satisfiable (who knows?); EJR is always satisfiable, but only in exponential time (unless $\texttt{P} = \texttt{NP}$). Can we get to poly-time computation?

Definition: Extended Justified Representation up to One Project (EJR-1)

A budget allocation π satisfies *EJR-1* if there exists no coalition $C \subseteq N$ for which there is $P \subseteq \bigcap_{i \in C} A_i$ such that $c(P) \leq b \cdot \frac{|C|}{n}$, and such that for all agents $i \in C$, and for all projects $p \in A_i$, we have:

$$sat_i^{cost}(P) > sat_i^{cost}(\pi \cup \{p\}).$$

Not proved: We can always compute in polynomial-time a budget allocation satisfying EJR-1.

Peters, Pierczynski, and Skowron "Proportional Participatory Budgeting with Additive Utilities" (2021)

Strategy-Proofness and Proportionality

Strategy-proofness for F: No agent $i \in N$, with true preferences $A_i^\star \subseteq \mathcal{P}$, can submit $A_i' \subseteq \mathcal{P}$, s.t.

$$sat_i^{cost}(F(A_i', \mathbf{A}_{-i})) > sat_i^{cost}(F(A_i^\star, \mathbf{A}_{-i})).$$

There exists no mechanism for approval-based PB that can satisfy both strategy-proofness and EJR.

The exact result is stronger (and requires some basic assumptions):

- It also applies when all projects cost 1;
- It also applies to much weaker axioms than EJR.

This result is proved using the SAT solver approach: we prove a base case using a SAT solver, and prove an induction lemma showing that the result extends above the base case.

Peters "Proportionality and Strategyproofness in Multiwinner Elections" (2018)

4. Conclusion

Wrapping Up

Today we have:

- Introduced participatory budgeting, specifically approval-based PB;
- Discussed how to use judgment aggregation to talk about PB;
- Investigated fairness properties for PB.

Participatory budgeting is a hot topic in computational social choice that is evolving quite fast. There is a lot of other directions that have been explored, and much more to come!

The rest of the course will be dedicated to the projects (more or less). Do not forget to submit your 2-page draft by *19:00 on Tuesday the 11th*.