Coursework 1

Deadline: Friday, 22 February 2002, 4pm (Room 539)

How to answer questions like this. The following questions are partly 'mechanical' (like proofs in KE or Resolution or transformation into a normal form, etc.) and partly require some original thinking. In both cases, make sure you clearly understand what the question is, even for the mechanical ones. As a general rule (which applies both here and for the exam), make sure the person marking your work has a chance to understand what you did or tried to do. In the case of proofs in one of the deduction systems, for example, you should indicate which rules and formulas you used in each proof step.

Question 1. Use KE to show that the following set of formulas is not satisfiable.

$$\{(\exists x)(P \rightarrow Q(x)),\ (\exists x)(Q(x) \rightarrow P),\ (\forall x)(\neg P \leftrightarrow Q(x))\}$$

Question 2. Show that eliminating a double negation is a *sound* KE rule, that is, prove that applying the double negation rule to a satisfiable branch in a KE tableau always yields another satisfiable branch.

Question 3. Show that the following formula is satisfiable by giving a model in which it is true. Derive the model using KE and show your working.

$$(\exists x)R(a,x) \wedge (\forall x)(\forall y)(R(x,y) \rightarrow \neg R(y,x))$$

Question 4. We have not explicitly stated the eta rules for Smullyan's Tableaux calculus in the course. Formulate them yourself! (*Hint:* You need to give Tableaux rules for formulas of the forms $A \leftrightarrow B$ and $\neg(A \leftrightarrow B)$. These rules should *not* require any minor premises.)

Question 5. Compute a most general unifier (mgu) of the following set of literals. Use Robinson's Unification Algorithm and show your working.

$$\{Q(f(x,g(x,a)),z),\ Q(y,h(x)),\ Q(f(b,w),z)\}$$

Question 6. Compute a Skolem Normal Form for the following formula. (*Hint:* First make sure that variables are named apart and then compute the Prenex Normal Form. Finally, Skolemise and transform the matrix into CNF.)

$$(\forall x)(\exists y)[P(x,g(y)) \rightarrow \neg(\forall x)Q(x)]$$

Question 7. For every man there is a woman who loves him. There is no woman who loves Al. Therefore, Al is not a man. Use the free-variable version of Smullyan's Tableaux calculus to prove:

$$(\forall x)(M(x) \rightarrow (\exists y)(W(y) \wedge L(y,x))),\ \neg(\exists x)(W(x) \wedge L(x,a)) \models \neg M(a)$$

Question 8. Transform the following Prolog program into a set of first-order clauses.

```
father( X, Y) :- parent( X, Y), male( X).
grandfather( X, Z) :- father( X, Y), parent( Y, Z).
parent( bob, sue).
parent( sue, mary).
male( bob).
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Use Resolution to show that the following query will succeed given the above program. What are the answer substitutions?

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?- parent( X, Y), grandfather( bob, Y).
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