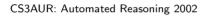# Free-variable Tableaux

One of the main drawbacks of KE and Smullyan's Tableaux, as we've seen them so far, is that for every application of the gamma rule we have to *guess* a good term for the substitution.

Now that we have learned about unification, we can devise better systems that will allow us to 'postpone' the decision of what substitution to choose until we attempt to close branches.

Instead of substituting with ground terms we will use free variables. Hence, the names *free-variable Tableaux*. (We will not discuss free-variable KE in this course.)
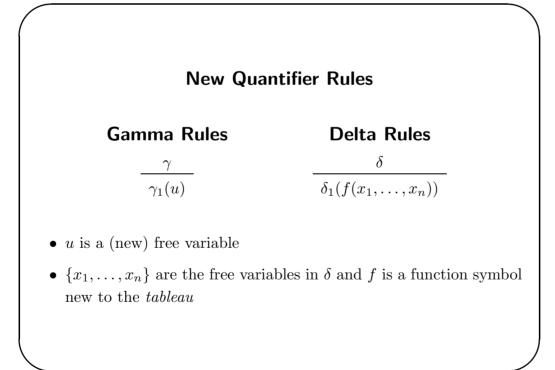
Propositional rules remain the same as before.

New Quantifier Rules

Gamma Rules

$$\frac{\gamma}{\gamma_1(u)}$$

Delta Rules

$$\frac{\delta}{\delta_1(f(x_1, \ldots, x_n))}$$

- u is a (new) free variable

- $\{x_1, \ldots, x_n\}$ are the free variables in δ and f is a function symbol new to the *tableau*

Closing Branches by Unification

The new closure rule reads as follows:

$$\frac{\begin{array}{c} A \\ \neg B \end{array}}{\times}$$

if there is a substitution σ such
that $\sigma(A) = \sigma(B)$

The closing substitution σ has to be *the same* for every branch on
the tree. Therefore, it is useful to choose as σ the mgu of A and B,
as this will be the least restrictive.