Tutorial 1

Question 1. Suppose a newspaper announces the following competition:

> *Every reader may send in a (rational) number between* 0 *and* 100*. The winner is the player whose number is closest to* $\frac{2}{3}$ *times the arithmetic mean of all submissions (in case of a tie the prize money is split equally amongst those with the best guesses).*

(a) Describe some strategies that people might use to decide what number to send in.

(b) Does this game have a Nash equilibrium? If yes, what is it?

(c) What changes about Nash equilibria if players can only choose integers?

(d) What changes if players can only choose integers and the mean is being multiplied by $\frac{9}{10}$ rather than $\frac{2}{3}$?

Question 2. Recall the definitions of *Pareto optimality* and *(utilitarian) social welfare* given in the lecture and answer the following questions:

(a) Is it possible that a Pareto optimal agreement does not have maximal social welfare? If yes, give an example; if no, explain why not.

(b) Is it possible that an agreement that maximises social welfare is not Pareto optimal? If yes, give an example; if no, explain why not.

Question 3. Consider the following instance of the Game of Chicken:

u_A/u_B	B defects	B cooperates
A defects	0/0	8/1
A cooperates	1/8	5/5

(a) There are two Nash equilibria with pure strategies for this game. What are they?

(b) These pure equilibrium strategies are not very attractive in practice. Explain why.

(c) There is also a Nash equilibrium with mixed strategies, where each agent either defects or cooperates with a certain probability. Compute this mixed Nash equilibrium (that is, compute the probabilities).

Hint: Start by putting yourself in the position of agent A and let p be the probability that A will defect. It is in the interest of A to choose p such that neither of the two pure strategies available to B would definitely be the better choice. What value of p should A choose?