Duncan Black - *On the rationale of group decision-making*

Charlotte Vlek

June 4, 2009

Table of contents

Background

- Black was a 'founding father' of social choice theory (Tullock 1991)
- "Duncan Black essentially rediscovered ideas that had been advanced earlier by the two 18th centory French noblemen [Compte de Borda and Marquis de Condorcet] only to be lost, then to be rediscovered late in the nineteenth century (1884) by Charles Dodgson (Lewis Caroll), then to be lost again. Since Black's discovery has not been lost, he must be viewed as the true founder of public choice (Rowler 1991 in Rowler, Schneider 2004 p. 203)
- Black: "(...) there is no part of economic theory which applies" (Black 1948, p. 23)

Blacks paper - overview

An overview:

- Situation:
  - members of a committee vote for motions or candidates from a given set
  - each voter can make a definite ranking
- Notation:
  - straight lines to represent preferences
  - graph
- Case 1: single-peaked curves as preferences
  - where do single-peaked curves occur
  - theorem about single-peaked curves
- Case 2: non single-peaked curves
  - discussion of problems
  - possible solution

Blacks paper - implicitly mentioned

- Voting rule: a Condorcet method (with no solution in case of ties)

  > *"What we are looking for is that motion which can defeat every other by at least a simple majority" (p. 26)*

- Manipulability is not an issue (yet)

  > *"(...) it is reasonable to assume that, when these motions are put against each other, he [the voter] votes in accordance with his valuation" (pp. 23-24)*

Occurence of single-peaked curves

"there is reason to expect that, in some important practical problems, the valuation actually carried out will tend to take the form of isolated points on single-peaked curves" (Black 1948, p. 24)

- Numerical quantities such as legal school-leaving age, etc.
- Motions and amendments
- What about voting for candidates?

Assumptions

- *m* motions
- Each motion is put against every other (in practice not necessary with transitivity)
- Final decision: that motion, if any, which is able to get a simple majority over every other

First note that:

> *There can be at most one motion with a simple majority over every other.*
> *(There is at most one Condorcet winner)*

(proof by contradiction)

Black's Theorem

- **Theorem**: If all voters have single-peaked curves as preferences, then the median motion will be adopted by the committee (Black 1948, p. 27)
- **Median Voter Theorem**: If x is a single-dimensional issue and all voters have single-peaked preferences defined over x, then x_m, the median position, cannot lose under majority rule (Mueller 2003, p.86)

Black's proof

- Suppose $a_1, ...a_m$ are the motions to vote for
- Let O_i for $0 < i \leq n$ be a numbering of the peaks such that $O_j \leq O_k$ if $i \leq k$ (without loss of generality, we can re-enumerate the voters such that voter x corresponds to peak O_x)
- Observe that for any voter x, for all motions $a_k \leq a_j \leq O_x$, he will prefer (or be indifferent) a_k over a_j
- Similarly, for any voter x for all motions $a_k \geq a_j \geq O_x$, he will prefer (or be indifferent) a_j over a_k

Black's proof

- Suppose n is odd. Then the median peak is $O_{\frac{n+1}{2}}$.
  - For any $a_k < O_{\frac{n+1}{2}}$, at least $\frac{n+1}{2}$ voters (all those with peaks right of the median peak and the median voter itself) will prefer (or be indifferent) $O_{\frac{n+1}{2}}$ over a_k.
  - For any $a_k > O_{\frac{n+1}{2}}$, at least $\frac{n+1}{2}$ voters (peaks left of the median peak and median peak itself) will prefer (or be indifferent) $O_{\frac{n+1}{2}}$ over a_k.

  So for every a_k, a majority prefers $O_{\frac{n+1}{2}}$ over it.
- Suppose n is even. Then at most there can be a tie between $O_{\frac{n}{2}}$ and $O_{\frac{n}{2}+1}$ and a chair must decide in the end.

Consequences

- Similarity with economics: actual shape of curves has no influence
- No voter or group of voters can alter their voting to make a motion more preferred by them be adopted instead
- Transitivity of the voting

Non single-peaked curves

- Represent votes in matrix
- If a Condorcetwinner exists, it is again non-manipulable and transitive (similar proofs)
- But "no motion need exist which is able to get at least a simple majority over every other" (Black 1948, p. 32)
  "this is by no means exceptional" (Black 1948, p.33)
- The voting might not be transitive anymore

Black's goals

Blacks main point seems

- *not* to discuss the best voting rule
- to find a method to quickly determine the winner in case of
  - single-peaked curves (the median peak)
  - non-single-peaked curves (a [Condorcet] winner might not exist)

Black's goals

For us, the main result is not how to find the winner, but that a given single-peaked curves, a Condorcet winner exists.

Or, as presented in the lecture slides:

> *"On single-peaked domains, social choice works very well:*
> *the Condorcet Paradox, Arrows Theorem, and the Gibbard-Satterthwaite Theorem all go away."*

Discussion

- What was Black's message? (Absence of Condorcet paradox? Median Voter Theorem?)
- In what cases will the curves be single-peaked? How useful is this theorem?
- What about applicability to candidates instead of motions?

Literature

- Tullock, G., 1991. *Duncan Black, the founding father*, Public Choice 71, pp. 125-128.
- Rowley, C.K; Schneider, F., 2004. *The encyclopedia of public choice*. Springer.
- Mueller, D.C., 2003. *Public choice III*. Cambridge University Press, Cambridge.