UvA Matching 2017: Problem Solving with Prolog (BSc KI)

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[https://staff.science.uva.nl/u.endriss/teaching/uva-matching/]

Plan for this Module

- Big picture:
  - What is *Artificial Intelligence* (AI)?
  - What is *problem solving* and why is it so central to AI?
  - How can *logic programming* help?
- And then for the actual topic of this lecture:
  - Strategies for solving *Towers of Hanoi*
  - Implementing those strategies in (simplified!) *Prolog*

Artificial Intelligence

What is AI? People do not agree on how to answer this.

My favourite answer:

> *AI is whatever AI researchers do.*

This is true, but not helpful to you right now. So maybe this:

> *AI is about getting machines to perform tasks that we tend to associate with "intelligence" when performed by humans.*

More specifically:

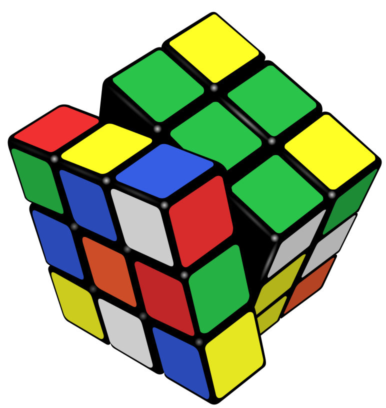
- no need for the machine to *be* intelligent (philosophy)
- no need to agree what "intelligence" *is* exactly (psychology)
- no need to focus *only* on things humans are good at

Problem Solving

AI is about getting machines to perform tasks ...

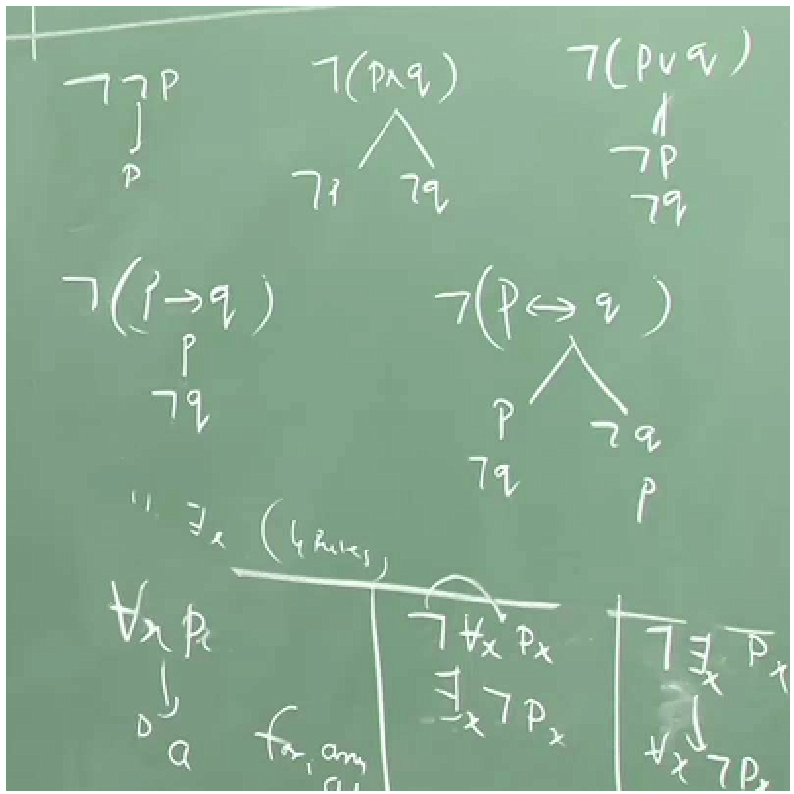
At the core of such (possibly physical) tasks there often are rather *abstract problems* we need to solve. A *solution* often requires us to find a sequence of *actions* to achieve a given *goal:*

- finding a path from A to B on a street map
- computing a sequence of electronic signals for a robot to do X
- solving Rubik's Cube



Logic Programming

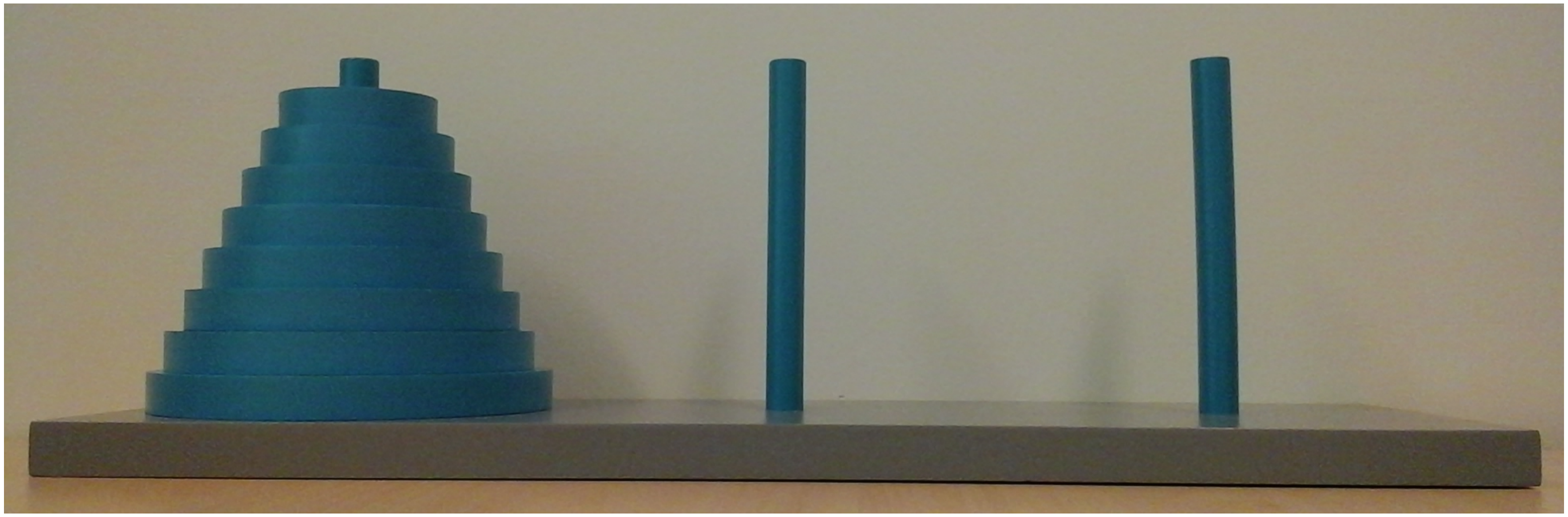
Prolog is a programming language that is particularly useful for problem solving. It is based on an idea called *logic programming*.



Towers of Hanoi

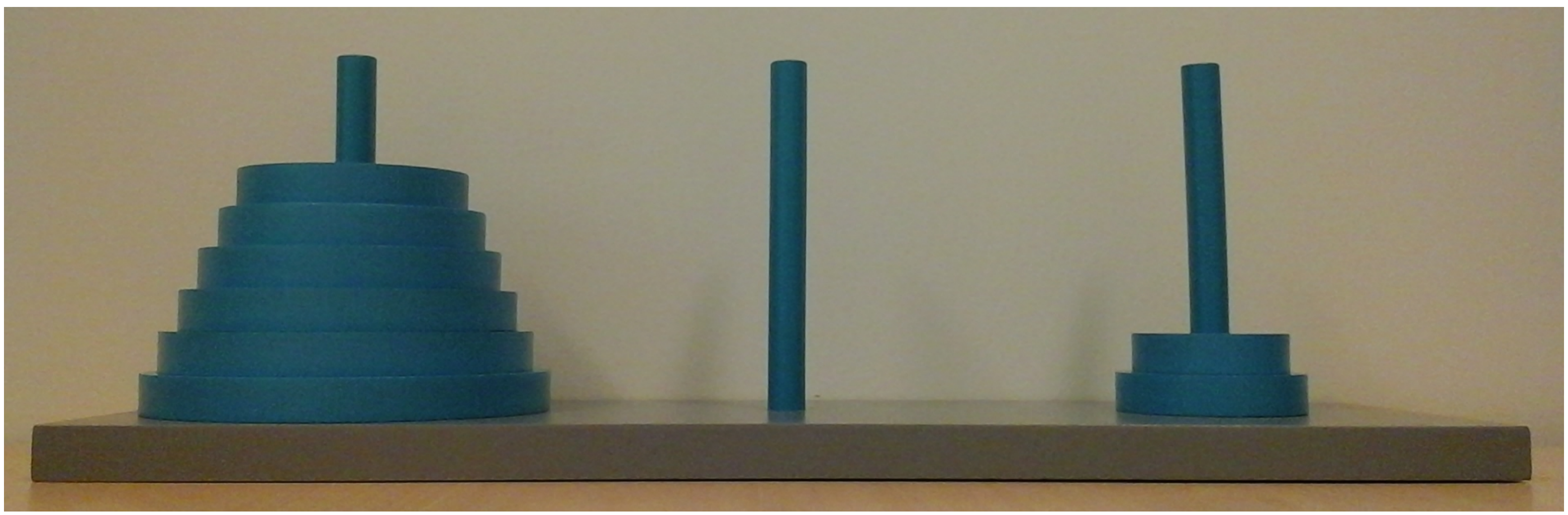
Move all disks from the leftmost to the rightmost peg, whilst:

- moving only one disk at a time, and
- never placing a disk on top of a smaller disk



Problem Representation

We don't want to build a robot that can physically move the disks (not today, anyway). We just want to find a *general method* that tells us in which order to move the disks.



First, we need to find an appropriate level of *representation* . . .

Graphical Representation

```
           | |                         | |                        | |
           | |                         | |                        | |
           | |                         | |                        | |
       XXXXXXXXXXX                     | |                        | |
      XXXXXXXXXXXXX                    | |                        | |
     XXXXXXXXXXXXXXX                   | |                        | |
    XXXXXXXXXXXXXXXXX                  | |                        | |
   XXXXXXXXXXXXXXXXXXX                 | |                      XXXXXXX
  XXXXXXXXXXXXXXXXXXXXX                | |                     XXXXXXXXX
================================================================================
            1                           2                          3
```

Prolog-internal Representation

```
Game = [[3, 4, 5, 6, 7, 8, 9], [9], [1, 2, 9]]
```

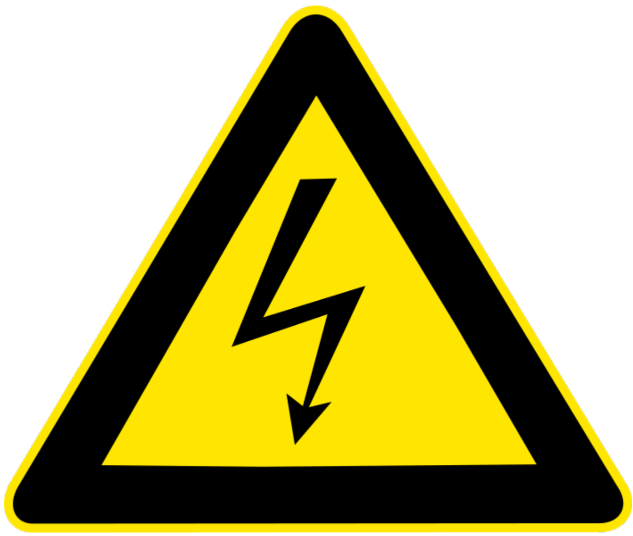
Representation in Terms of Bits

```
01001010 11010010 11101010 0010111 10100101 00101010
11101010 00101100 00001000 1101010 11010101 11010101
00101010 01011101 11010101 0010101 10100110 10100100
00101011 10100111 01011111 1010010 10101110 10101110

10000010 10101000 00101010 0001101 11010010 00001011
00010100 10001001 11101000 0111101 01010110 11110010
00101110 10100010 11010100 1010100 01111011 11001000
11110101 10100010 00110101 1101000 11100000 11000011

01110100 01001100 10100010 1110010 00101010 11110101
00101011 11101010 00001100 0101010 10101011 11100101
01010001 01010100 11100001 0101011 11110101 11000010
11101000 11100010 00101110 1010100 11000000 10001011
```

Physical Representation on the Computer



Game Console

You cannot learn how to program in just one day. And anyway, we want to focus on *high-level ideas*, not on low-level nitty gritty.

All the low-level stuff is taken care of by the *ToH Game Console*, a Prolog program providing a few simple *predicates* ("commands") for you to manipulate the towers.

`init(+N)`	create a game with `N` disks (with $\mathtt{N} \in \{1, 2, 3, 4, \ldots\}$)
`move(+A,+B)`	move the top disk on peg `A` to `B` ($\mathtt{A}, \mathtt{B} \in \{1, 2, 3\}$)
`top(+A,-D)`	given peg `A`, return top disk `D` on `A` (if any)
`empty(+A)`	check whether peg `A` is empty

Here '`+`' means that you *provide* a value and '`-`' means that you *receive* a value (by using a variable). Example:

```
?- top(1,X).
X = 3.
```

First Solution

Focus on the case of *3 disks* for now.

```
To solve the game, you have to
  move the top disk on peg 1 to peg 3, and then
  move the top disk on peg 1 to peg 2, and then
  move the top disk on peg 3 to peg 2, and then
  ...
```

In Prolog this is implemented as follows:

```
sillySolve :-
  move(1,3), move(1,2), move(3,2), move(1,3),
  move(2,1), move(2,3), move(1,3).
```

This will work. Type into Prolog (with the Game Console loaded):

```
?- init(3), sillySolve.
```

Good solution?

Second Solution

What are the possible moves available at a given time?

- move *disk 1* (the smallest disk) "to the *right*" [1R]
- move *disk 1* (the smallest disk) "to the *left*" [1L]
- make the only possible move *not involving disk 1* [N1]

  (note that N1 is not possible in the initial or final configuration)

Observations:

- never iterate 1R/1L (can achieve same effect with one move)
- never iterate N1 (amounts to undoing previous move)

Thus:

- should alternate 1R/1L with N1

But which one, 1R or 1L? Leap of faith: let's just try with 1R.

The Iterative Solution

To summarise, our algorithm is this:

(1) Perform 1R (move disk 1 to the right, from peg 1 to peg 2).

(2) *Repeat* until the final configuration is reached:

  (a) Perform N1 (only possible move not involving disk 1).

  (b) Perform 1R (move disk 1 one peg "to the right").

This works! (at least when the number n of disks is even)
If n is odd, use 1L instead of 1R.

This is pretty cool. But hard to understand *why* it actually works.

The Iterative Solution in Prolog

This is not so easy to implement and we'll brush over some details.

```
move1R :-                  % to perform 1R:
  where(1,Peg),            % find peg of disk 1
  right(Peg).              % move disk 1 one peg to the right

moveN1 :-                  % to perform N1:
  where(1,Peg1),           % find peg of disk 1
  whereOtherDisk(Peg2),    % find peg of next smallest top disk
  getThirdPeg(Peg1,Peg2,Peg3), % find name of third peg
  move(Peg2, Peg3).        % move second smallest disk to third peg

itSolve :-                 % to solve the game:
  move1R, finish.          % perfom 1R, then finish off

finish :- empty(1), empty(2). % finished if: pegs 1 & 2 empty

finish :-                    % to finish:
  moveN1, move1R, finish.    % perform N1, then 1R, then finish off
```

Third Solution

<u>Wanted:</u> method to move a tower of *n disks* from *peg* `A` to *peg* `B`.

<u>Solution:</u>

To move a tower of n (with $n > 1$) *disks* from `A` to `B`, simply

(1) move $n-1$ disks from `A` to `C`,

(2) move the largest disk from `A` to `B`, and

(3) move $n-1$ disks from `C` to `B`

For the case of a tower with just *1 disk*, we know what to do.

This works. Magic?

Two Auxiliary Predicates

We will need two auxiliary predicates that take care of some very basic stuff for us that is related to the manipulation of numbers.

`getPredecessor(+N,-P)`: Given a natural number `N`, return that number's predecessor $\mathtt{P} := \mathtt{N} - 1$. Example:

```
?- getPredecessor(10,X).
X = 9.
```

`getThirdPeg(+A,+B,-C)`: Given the names of pegs `A` and `B`, return that of the third peg `C` (names are `1`, `2`, and `3`). Example:

```
?- getThirdPeg(3,1,X).
X = 2.
```

Btw, all that `getThirdPeg/3` does is to compute $\mathtt{C} := 6 - (\mathtt{A} + \mathtt{B})$.

The Recursive Solution in Prolog

`recSolve(N,A,B)` implements our recursive algorithm to move a tower with `N` disks from location `A` to location `B`:

```
recSolve(N, A, B) :-         % to move a tower from A to B
  N =:= 1,                   % if size N is equal to 1:
  move(A, B).                % move single disk from A to B

recSolve(N, A, B) :-         % to move a tower from A to B
  N > 1,                     % if size N is greater than 1:
  getPredecessor(N, N1),     % get predecessor N1 of N
  getThirdPeg(A, B, C),      % get name of third peg C
  recSolve(N1, A, C),        % move tower of size N-1 to C
  move(A, B),                % move largest disk to B
  recSolve(N1, C, B).        % move tower of size N-1 to B
```

Complexity

How complex is the *Towers of Hanoi* problem? That is, *how long* does it take to solve a game with *n disks*?

<u>Answer:</u> depends how fast you can move one disk

<u>So better ask:</u> *how many steps* to solve a game with *n disks*?

- $n = 1$: ?
- $n = 2$: ?
- $n = 3$: ?
- ...

Formal Complexity Analysis

Observations: You must move the largest disk (size n) eventually. You can only move it, if the remaining tower sits somewhere else. So our recursive algorithm is optimal:

(1) move tower of size $n{-}1$ out of the way
(2) move largest disk in place
(3) move tower of size $n{-}1$ on top of it

Let $f(k)$ be the *number of steps* needed for a game with *k disks*.

We know $f(1) = 1$ (solving a game with 1 disk takes 1 step).

From our algorithm, for a game with *n disks* we immediately get:

$$f(n) \;=\; f(n{-}1) + 1 + f(n{-}1) \;=\; 2 \cdot f(n{-}1) + 1$$

Not yet helpful for computing, say, $f(10)$. We want a *closed form*.

Closed Form

What is $f(n)$, the *number of steps* to solve a game with n *disks*?

We have established:

- $f(1) \;=\; 1$
- $f(n) \;=\; 2 \cdot f(n{-}1) + 1 \quad (*)$

Examples: $f(2) = 3$, $f(3) = 7$, $f(4) = 15$, $f(5) = 31$, $f(6) = 63$, ...

What we really want:

Claim: $f(n) = 2^n - 1$ for every $n \in \mathbb{N}$.

Proof: Take any $n > 1$. *Suppose* for a moment that the claim were true for $n{-}1$, meaning that $f(n{-}1) = 2^{n-1} - 1$. $(**)$ But *then*:

$$f(n) \;\stackrel{*}{=}\; 2 \cdot f(n{-}1) + 1 \;\stackrel{**}{=}\; 2 \cdot \left[2^{n-1} - 1\right] + 1 \;=\; 2^n - 1$$

The claim is true for $n = 1$, as we have seen.

Thus, *by induction*, it must be true for *every* $n \in \mathbb{N}$. ✓

Summary

What you've learned about today:

- several methods for solving *Towers of Hanoi*
- programming in *Prolog* (well, a bit)

What this really has been about:

- need to find the right *representation* of a problem
- need to *abstract* away from low-level details
- importance of striving for *general* solutions
- importance of formal/mathematical *analysis* of solutions

What next?

- grab a copy of the *handout* on your way out (to have *lunch*)
- if not done yet, install *SWI-Prolog* on your laptop
- be at one of the lab rooms at 14:00 for the *practicum*
- submit your *assignment* by 17:00 today