Restricted Manipulation in Iterative Voting

Umberto Grandi

Department of Mathematics
University of Padova

11 April 2013

Joint work with Andrea Loreggia, Francesca Rossi,
K. Brent Venable and Toby Walsh

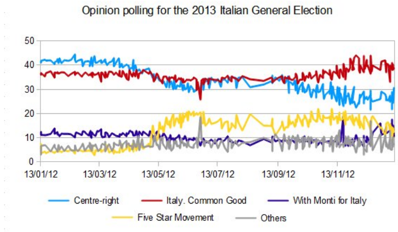
Good Manipulation, Bad Manipulation

Manipulation in elections is usually considered a bad thing, to be avoided or at least to be made computationally difficult to achieve.

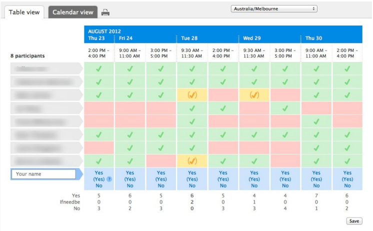
What if we can get a better outcome with iterated manipulation of simple rules, rather than complex-information-costly-almost-strategy-proof rules?

Practical Examples

In practice, iterative manipulation do occur:



Iterative response
to repeated polls



Approval voting with
iterative manipulation

Image source: Wikipedia, Doodle.com

Outline

1. The setting:
   - Voting rules (in brief)
   - Iterative voting
   - Restricted manipulation: *M1* and *M2*

2. Theoretical evaluation
   - Convergence: Yes! (unknown for STV)
   - Axiomatic properties: transfer to iterative rules

3. Experimental evaluation
   - Condorcet efficiency: Increase!
   - Average position of the winner: Increase!

Voting Rules

Things you all know:

- We start from a profile of linear orders over candidates $\{c_1, \ldots, c_m\}$.
- **Positional Scoring Rules** give s_j points to candidates in position j in individual preferences, and elect the candidates with maximal score. We consider: Plurality, Borda, 2-approval, 3-approval, veto.
- **Copeland** elects the candidates which maximise the number of pairwise comparisons won minus the number of pairwise comparisons lost.
- **Maximin** elects the candidates with the highest minimal number of voters preferring her in pairwise comparisons.
- **Single Transferable Vote** deletes the candidate with the least first positions in individual preferences, transfer the votes to the succeeding candidate, and iterates until there is one candidate which has the majority of first positions.

Assumption: linear tie-breaking (for these slides $a > b > c > \ldots$)

Strategic Manipulation

Manipulation occurs whenever a voter changes her ballot in her favour:

$$
\begin{array}{c} a \succ b \succ c \\ b \succ c \succ a \\ c \succ b \succ a \\ \hline \text{Plurality: } a \end{array}
\quad \longrightarrow \quad
\begin{array}{c} a \succ b \succ c \\ b \succ c \succ a \\ \color{red}{b} \succ \color{red}{c} \succ a \\ \hline \text{Plurality: } b \end{array}
$$

Strategic Manipulation

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$a \succ b \succ c$ $b \succ c \succ a$ $c \succ b \succ a$	$\longrightarrow$	$a \succ b \succ c$ $b \succ c \succ a$ $b \succ c \succ a$
Plurality: a		Plurality: b

Is there any chance to avoid manipulation?

Theorem [Gibbard-Satterthwaite]

Given a voting rule F, one of the following facts must be true: (i) there is a candidate that never wins (ii) F is a dictatorship, (iii) F can be manipulated.

Needless to say, all voting rules presented are manipulable...

A. Gibbard. Manipulation of voting schemes: A general result. *Econometrica*, 1973.

M. A. Sattertwaithe, Strategy-proofness and Arrows conditions... *JET*, 1975.

Voting Games / Iterative Voting

Strategic manipulation in elections defines a voting game:

- Strategies are linear orders: individuals can change their preferences to obtain a better outcome
- The outcome is the result of the voting rule
- Utilities are defined by the truthful preferences of individuals

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Definition

Given a set of manipulation moves M, a voting rule F (and a turn function) the iterated voting rule F^M associates with every profile $\mathbf{b}$ the outcome of convergent iteration of manipulation moves in M (or $\uparrow$ if it does not converge).

Unrestricted manipulation does not always converge! But if it does, it converges to a Nash equilibrium of the voting game associated to F.

R. Meir Et Al. Convergence to equilibria in plurality voting. AAAI-2010.

O. Lev and J. S. Rosenschein. Convergence of iterative voting. AAMAS-2012.

Restricted Manipulation

Manipulation moves studied in the literature:

- **Best response** (no restriction): choose the ballot that changes the outcome of the election in the best way.
- **k-pragmatist**: put in first position your favourite candidate among the top k in the outcome of the voting rule.

A. Reijngoud and U. Endriss. Voter response to iterated poll information. AAMAS-2012.

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How to evaluate a restriction on manipulation moves?

Convergence	**Computation**	**Information**
Guaranteed	Not costly	Low
(small number of steps)	(not NP-hard!)	(top candidate, scores..)

Restricted Manipulation: *M1*

Iteration starts at $\mathbf{b}^0$ (truthful) and continues to $\mathbf{b}^1, \ldots, \mathbf{b}^k$ until convergence

M1

Move to the top the second-best candidate in b_i^0 (truthful), unless the current winner $w = F(\mathbf{b}^k)$ is already her best or second-best candidate in b_i^0 (truthful)

$$\begin{array}{c} a \succ b \succ c \succ d \\ c \succ b \succ a \succ d \\ d \succ b \succ c \succ a \\ \hline \text{Plurality: } a \end{array} \quad \rightarrow \quad \begin{array}{c} a \succ b \succ c \succ d \\ b \succ c \succ a \succ d \\ d \succ b \succ c \succ a \\ \hline \text{Plurality: } a \end{array} \quad \rightarrow \quad \begin{array}{c} a \succ b \succ c \succ d \\ b \succ c \succ a \succ d \\ b \succ d \succ c \succ a \\ \hline \text{Plurality: } b \end{array}$$

Minimal computation cost, minimal information required.

A side note: b is the Condorcet winner.

Restricted Manipulation: *M2*

M2

move to the top the best candidate in b_i^0 (truthful) which is above $w = F(\mathbf{b}^k)$ in b_i^k (reported), among those that can become the new winner of the election

$$
\begin{array}{c}
\mathbf{a} \succ b \succ c \succ d \\
b \succ c \succ \mathbf{a} \succ d \\
d \succ \mathbf{a} \succ b \succ c \\
c \succ d \succ b \succ \mathbf{a} \\
\hline
\text{Plurality: } a
\end{array}
\rightarrow
\begin{array}{c}
a \succ b \succ \mathbf{c} \succ d \\
c \succ b \succ a \succ d \\
d \succ a \succ b \succ \mathbf{c} \\
\mathbf{c} \succ d \succ b \succ a \\
\hline
\text{Plurality: } c
\end{array}
\rightarrow
\begin{array}{c}
a \succ b \succ c \succ d \\
c \succ b \succ a \succ d \\
a \succ d \succ b \succ c \\
c \succ d \succ b \succ a \\
\hline
\text{Plurality: } a
\end{array}
$$

Low computation cost, low information required (score, majority graph).

Convergence

Theorem

F^{M1} converges for every voting rule.

Proof idea: *M1* can be applied only once by each individual.

Theorem

F^{M2} converges for PSR, Copeland and Maximin.

Proof idea: the score of the winner increases at every step, or remains the same and the candidate moves up in the tie-breaking order.

Axiomatic Properties

Axiomatic properties are preserved at every step of the iteration:

Theorem

M1 and M2 preserve unanimity.

If we start from a unanimous profile, the winner is always the top preferred candidate at every step of the iteration.

Theorem

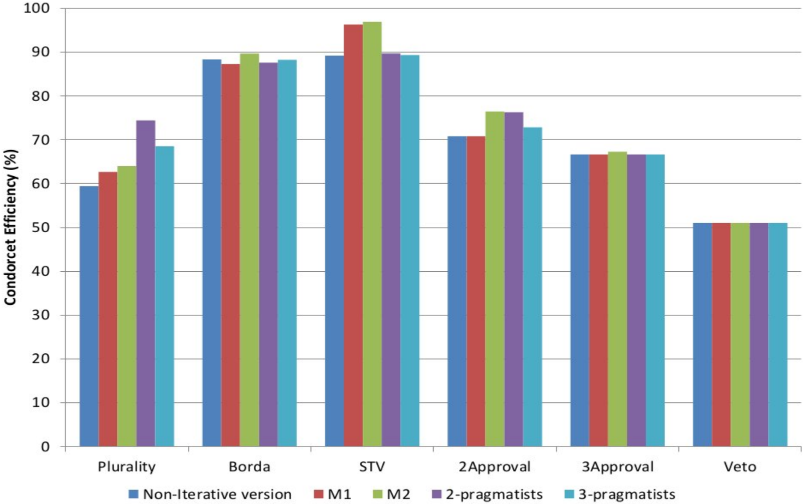
M1 and M2 preserve Condorcet consistency.

Same for anonymity and neutrality. Pareto-condition does not transfer.

Condorcet Efficiency I

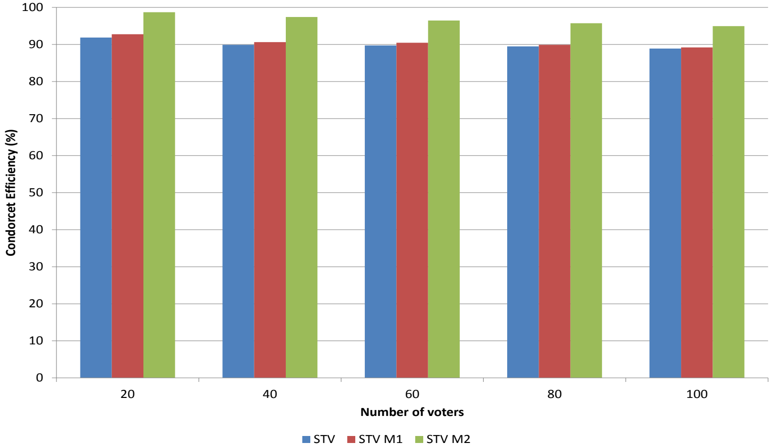
Disclaimer: We used impartial culture assumption!

For Plurality better 2P and 3P, for all others *M2* is better.
Positive performance of *M1*, even if little changes.



Condorcet Efficiency II

Higher efficiency for $n = 20$, stabilizes at around $n = 60$.
Consistently more than 95% for STV!



Motivational Intermezzo (*M2*)

One further motivation for iterated manipulation is that the Condorcet winner may be extracted without having to ask for the full profile.

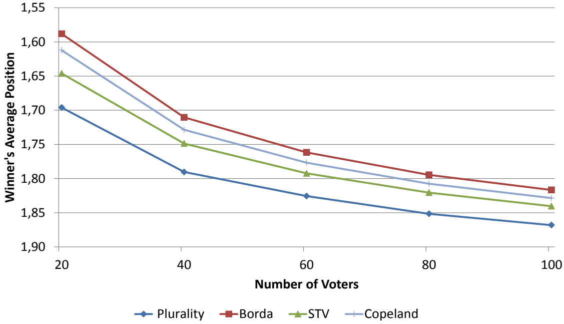
But: is it more costly to iterate or to ask for the full profile?

	# profiles with iteration	average # steps	maximal # steps
Plurality	2902	11.8	27
STV	1173	1.7	7
Borda	1961	8.1	31
2-Approval	2395	9.1	17

Profiles are 50×5, maximal number of iterations is 27: good for Plurality!
Iteration takes place between 10% and 30% of the cases:
Not very costly, given the increase in Condorcet efficiency!

Average Position of the Winner (aka Borda score)

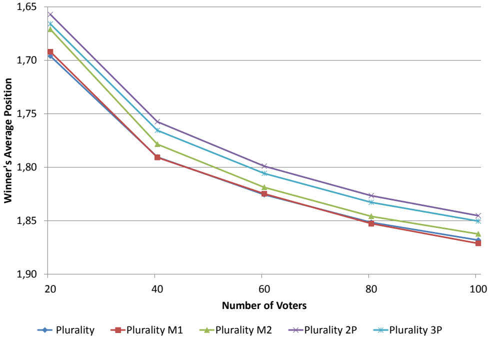
How much preferred is the winner in average?



Recall that Borda elects the candidate with the highest "average position"

Average Position of the Winner

For all voting rules (except for Borda) the position of the winner increases by allowing iterated restricted manipulation:



Conclusions and Future Work

We introduced two new restricted manipulation moves which are easy to compute and need small amount of information, and we evaluated:

- Convergence of restricted iterative voting
- Condorcet efficiency
- Average position of the winner (Borda score)
- Number of iteration steps

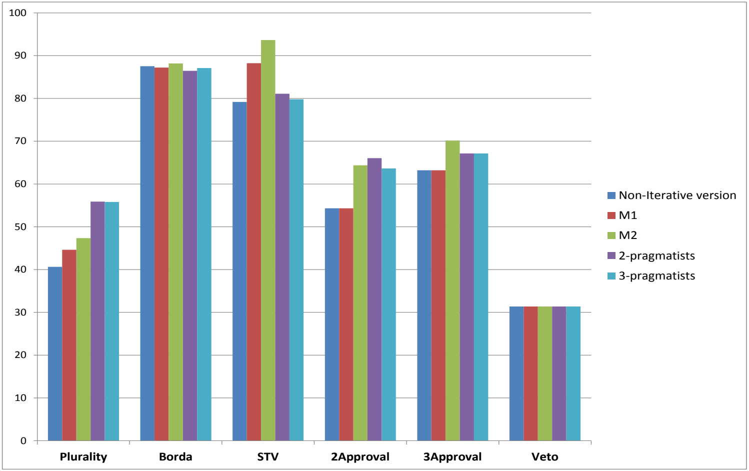
Restricted manipulation in iterative voting increases the Condorcet efficiency and the average position of the winner in a limited number of steps.

Lots of future questions:

- More realistic distribution of preferences (urn model, Mallow model).
- Other ideas for restricted manipulation move?
- Other parameters to evaluate performance of iteration?

Thank you for your attention!

Condorcet Efficiency



Condorcet Efficiency II

Higher efficiency for $n = 20$, stabilizes at around $n = 50$.