Voting Rules and Strategic Candidacy
Some recent results

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April 12, 2013

Example: Chosing the next Olympic city

The Olympic Commitee is about to decide the next olympic city.
4 cities are candidating: A'dam (a), Paris (p), Moscow (m), Bali (b).
The different cities have the following preferences regarding the result:

- $p : p \succ m \succ b \succ a$
- $a : a \succ m \succ b \succ p$
- $b : b \succ a \succ p \succ m$
- $m : m \succ p \succ b \succ a$

Plurality is used. The votes of the IOC are known (not unrealistic...).
Asterdam should be elected (4 points).

2	1	2	2	2
a	a	p	b	m
m	p	b	p	a
b	m	m	a	p
p	b	a	m	b

One before the deadline for candidacy, the four towns are candidates.

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Paris realize it cannot win, but it can prevent Amsterdam from being elected. Paris withdraws its candidacy (Bali should now be elected).

2	1	2	2	2
a	a	p	b	m
m	p	b	p	a
b	m	m	a	p
p	b	a	m	b

Two days before the deadline for candidacy, three towns are candidates.

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Amsterdam likes Moscow better than Bali.
Amsterdam withdraws its candidacy.

2	1	2	2	2
a	a	p	b	m
m	p	b	p	a
b	m	m	a	p
p	b	a	m	b

Moscow is the next Olympic city.

Outline of the Talk

The game-theoretical interpretation

We define a candidacy game as a normal form game $\Gamma = \langle X, P, r, P^X \rangle$ with m players, where:

- X is a set of candidates;
- P is a voters' profile;
- r is a voting rule (actually, a family of voting rules);
- P^X is a profile of the candidates' preferences

The strategy set available to each player is simply 1 (running for the election), or 0 (not running).

Note: We use $adf \mapsto d$ to say that candidate d wins in the (restricted) profile consisting of candidates adf.

B. Dutta, M. Le Breton, M. O. Jackson. *Strategic candidacy and voting procedures.* Econometrica-2001.

Assumptions

1. each candidate may choose to run or not for the election;
2. each candidate has a preference ranking over candidates;
3. each candidate ranks himself on top of his ranking;
4. the candidates' preferences are common knowledge among them;
5. the outcome of the election as a function of the set of candidates who choose to run is common knowledge among the candidates.

Research questions

Natural research questions include:

- for which rules can $(1, \ldots, 1)$ be guaranteed to be NE?
- for which rules can the existence of a NE be guaranteed?
- do natural (*e.g.* best response) dynamics converge?

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What is known so far...

- no non-dictatorial voting rule satisfying unanimity is candidacy-strategy-proof, that is $(1, \ldots, 1)$ cannot be guaranteed to be a NE.
- for voting trees, there are candidacy games with no NE.

B. Dutta, M. Le Breton, M. O. Jackson. *Strategic candidacy and voting procedures*. Econometrica-2001.

B. Dutta, M. Le Breton, M. O. Jackson. *Voting by successive elimination and voting procedures*. JET-2002.

Two preliminary remarks

First observe that with $m = 2$, $(1,1)$ is a NE (the winner does not leave by narcissim, and the other would not affect the outcome by leaving).

3 players case

For $m = 3$ players, any candidacy game has a NE

Idea. Assume that $(1,1,1)$ is not NE. Then one candidate has an interest to leave. But then the two remaining candidates must be in NE

Condorcet winner

For Condorcet-consistent rule, if c is the Condorcet winner, then any set X containing c is a (S)NE (and no other is).

Idea. As c remains a Condorcet winner in any subprofile, no agent $x \neq c$ has an interest to join or leave X. Obviously, c has no interest to leave (by the assumption of narcissim).

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Scoring rules

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We make use of a (powerful!) result by Saari:

> *For "almost" all scoring rules, any conceivable choice function can result from a voting profile*

- Our problem thus boils down to check whether there exists at least one feasible choice function which, taken along with some candidates' preferences, exhibits no NE in our candidacy setting.

D. Saari. *A dictionary of voting paradoxes*. JET-1987.

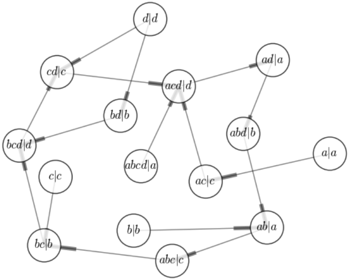
Scoring rules

The ILP encoding for choice functions must ensure that:

- in any state, there is be one winner;
- in any state, one agent at least must deviate to another state.

Regarding candidates' preferences we must ensure that:

- a candidate deviating must prefer the winner in the new state;
- preferences of candidates are transitive, irreflexive, narcissic.



Scoring rules

From one such feasible choice function, we can work out a profile for almost all scoring rules.

- For plurality and $m = 4$, there may be no NE.

3	1	1	1	1	1	1	2	2	a	b	c	d
d	d	d	a	a	a	b	b	c	a	b	c	d
c	b	a	b	c	d	c	a	b	b	a	d	a
a	c	b	c	b	b	d	c	d	c	c	a	b
b	a	c	d	d	c	a	d	a	d	d	b	c

... however Borda's voting rule stands out as an exception (basically, the sole exception) to Saari's result. (There is no such guarantee for Borda).

- How can we make sure that feasible choice functions can or cannot be rationalizable for Borda?

D. Saari. *A dictionary of voting paradoxes*. JET-1987.

Scoring rules: Borda

How can we deal with Borda?

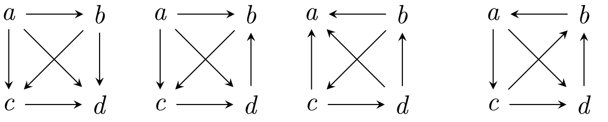
- the idea is to exploit the fact that the Borda rule can be represented by a weighted majority graph—and we try to construct a feasible weighted majority graph corresponding to the choice function;
- we include additional constraints into the ILP to account for the feasibility of a corresponding weighted majority graph.

$$\forall s \in S, \forall i \in A(s), \forall j \in A(s) \setminus \{i\} :$$
$$(1 - w_{s,i}) \times M + \sum_{j \in A(s) \setminus \{i\}} N_{i,j} \geq 1 + \sum_{j \in A(s) \setminus \{k\}} N_{k,j}$$

The infeasibility of the ILP tells us that Borda must always have a NE

Condorcet-consistent rules

We just assume Condorcet-consistency (CC).
Only 4 different tournaments, making case by case analysis possible.



G_1 G_2 G_3 G_4

- G_1 and G_2: a Condorcet-winner, any X containing a is NE
- G_3: Condorcet-loser a and a cycle $bcdb$. Wlog, $bcd \mapsto b$, but then take bc. No candidate wants to leaves, a does not join ($abc \mapsto b$ by CC), d does not join (since $bcd \mapsto b$). Hence bc is a NE.
- G_4: more tedious but can be shown in a similar manner.

Outline of the Talk

More candidates

Do counter-examples transfer to a larger number of candidates?

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- They do under a very mild assumption

Insensitiveness to bottom-ranked candidates (IBC)

Suppose a rule r elects x in a profile, then r must elect x if we add a new candidate at the bottom of every votes.

Satisfied by almost all voting rule (veto is an exception).

For any rule satisfying IBC, a profile without NE with n candidates can be extended to a profile with $n+1$ candidates without NE either.

- So any negative (no NE) result for m transfer to $m' > m$.

How about positive results?

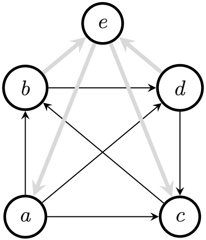
Condorcet-consistent rules

Now case analysis is not feasible any longer...

Copeland

For any number of candidates, and an odd number of voters, Copeland always has a NE.

Copeland winner a = tie-breaking winner among max. Copeland scores M_P the majority graph.
Set $Dom(a)$ = the set of candidates beaten by a in M_P.
Claim: $Y = \{a\} \cup Dom(a)$ is a NE.



a is Condorcet winner in Y.
No candidates wants to leave Y.

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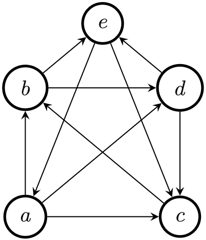

a is Condorcet winner in Y.
No candidates wants to leave Y.

Suppose some candidate $\notin Y$ joins.
But a maximizes the Copeland score (=|Dom(a)|).
The score of a is unaffected by y.
Other candidates can at best get the same score but would be beaten by tie-breaking...

Maxmin

Maybe the existence for a NE is guaranteed for any CC-rule?

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Maybe the existence for a NE is guaranteed for any CC-rule? No.

5 candidates suffice to show that NE are not guaranteed any longer.

Maximin

For maximin with $m = 5$ there may be no NE.

Take the following weighted majority graph:

	a	b	c	d	e	a	b	c	d	e
a	–	1	4	2	3	a	b	c	d	e
b	4	–	1	4	3	c	e	d	a	b
c	1	4	–	2	2	b	c	a	c	a
d	3	1	3	–	0	e	a	e	b	d
e	2	2	3	5	–	d	d	b	e	c

We have $abcde \mapsto e$, the maxmin winner. Furthermore, $abcd \mapsto a$, $abde \mapsto b$, $abce \mapsto e$, $acde \mapsto a$, $bcde \mapsto c$, ...

How about Strong Nash Equilibria?

k-NE allow deviation of k players simultaneously.
This is of course a much stronger requirement.

- already for 3 candidates the SNE existence result does not hold;
- for a larger number of candidates, we always could find examples without 2-NE (it takes only 2 agents to ruin stability).

Other remarks

- importance of narcissism—relaxing the constraint of narcissim can have a huge impact on the results
  Example for Borda, with 9 voters. Only b is not narcissic.

	a	b	c	d	a	b	c	d
a	–	5	6	3	a	c	c	d
b	4	–	8	5	b	d	a	a
c	3	1	–	6	c	b	d	b
d	6	4	3	–	d	a	b	c

- best response dynamics—even for those rules enjoying stability, we could exhibit cycles (no convergence) in best-responses dynamics.

Take-away message

- we studied a candidacy game introduced by Dutta et al.
- we exhibited a sharp contrast in the case of $m = 4$ candidates:
  - for almost all scoring rules, there may be no NE
  - for all CC-rules and Borda, the existence of NE is guaranteed
- the situation is less clear for more candidates, in particular:
  - there are CC-rules for which from 5 candidates already there may be no NE (maxmin)
  - there are CC-rules enjoying NE for any number of voters (*e.g.* Copeland, uncovered set)
- Complexity: insights can be gained by relating to control by adding/deleting candidates, but with consenting agents.