Walking a mile in your shoes: an Escape from Arrovian Impossibilities

Constanze Binder

Faculty of Philosophy
Erasmus Institute for Philosophy and Economics (EIPE)
Erasmus University Rotterdam

ILLC Workshop on Collective Decision Making
March 19-20, 2015
University of Amsterdam

The Literature

Social Choice Approach to Justice (Sen 2009)

- Comparative Approach
- Action-Guidance
- Facilitating Reexamination of Unquestioned Values & Convictions

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(How) Is the Social Choice Framework suited to address these points?

Outline

- The Social Choice Framework: Lessons from Existing Results
- Extending the Social Choice Framework
  - Procedure of Position Change
  - Position Change and a Domain Condition
  - Result: Value Overlap is sufficient for Action-Guidance
- Some Conclusions
- Open Questions & Future Research

The Social Choice Framework

- X finite set of alternatives
- R binary relation on X
- $\{1, \ldots, m\}$ set of individuals
- $(R_1, \ldots, R_m) \in \mathcal{R}^m$ profile of (strict) preference orderings
- $f : \mathcal{R}^m \to \mathcal{R}$

Example

R_1	R_2	R_3
x	x	x
y	y	y
z	z	z

$\longrightarrow$

R
x
y
z

Specification of ‘Action-Guidance’

What is required for ‘Action-Guidance’?
What are the necessary and sufficient conditions for R to induce a choice function?

- Optimization: Acyclicity and Completeness of R
- Maximization: Acyclicity of R

Insights of Existing Results in Social Choice Theory

- Impossibility of transitive and complete social ranking (Arrow 1953)
- Possibility of acyclic social ranking (Sen 1970)

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Problem

- Problem: social ranking cyclic and/or (highly) incomplete
- Escape Routes:
  - Domain Restrictions: Arbitrary?
  - 'Biting the Incompleteness Bullet':
    How convincing are the 'complete parts' (Weak Pareto)?
    Problem of Parochial Values!

Extending the Framework: Procedure of Position Change

Changing Perspectives: Extending the Framework

$d \in \mathcal{R}^m$	R_1	R_2	$\dots$	R_m	d^*
R_1	$R_{1,1}$	$R_{1,2}$	$\dots$	$R_{1,m}$	R_1^*
R_2	$R_{2,1}$	$R_{2,2}$	$\dots$	$R_{2,m}$	R_2^*
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
R_m	$R_{m,1}$	$R_{m,2}$	$\dots$	$R_{m,m}$	R_m^*

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$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
R_m	$R_{m,1}$	$R_{m,2}$	$\dots$	$R_{m,m}$	R_m^*

Implications for Acyclicity and/or Completeness of R?

Position Change: No Arbitrary Changes

$d \in \mathcal{R}^m$	xP_1y	xP_2y	xP_3y	d^*
xP_1y	$xP_{1,1}y$	$xP_{1,2}y$	$xP_{1,3}y$	xP_1^*y
R_2	$R_{2,1}$	$R_{2,2}$	$R_{2,3}$	R_2^*
R_3	$R_{3,1}$	$R_{3,2}$	$R_{3,3}$	R_3^*

1. For all $x, y \in X$, for all $i \in \{1, \ldots, m\}$, $xP_iy \& yP_i^*x \Rightarrow$ for some $j \in N$, yP_jx.

Position Change: Effective Empathy Outweighs Disagreement

$d \in \mathcal{R}^m$	R_1	xP_2y	yP_3x	d^*
xP_1y	$R_{1,1}$	$xP_{1,2}y$	$yP_{1,3}x$	yP_1^*x
R_2	$R_{2,1}$	$R_{2,2}$	$R_{2,3}$	R_2^*
R_3	$R_{3,1}$	$R_{3,2}$	$R_{3,3}$	R_3^*

1. For all $x, y \in X$, for all $i \in \{1, \dots, m\}$, $xP_iy \& yP_i^*x \Rightarrow$ for some $j \in N, yP_jx$.
2. For all $x, y \in X$, for all $i \in \{1, \dots, m\}$, $\#\{(x, y, i) \in X \times X \times \{1, \dots, m\} | xP_iy$ and $yP_i^*x\} >$ $> \#\{\{x, y\} \subseteq X|$ there is some $i, j \in \{1, \dots, m\}$ such that xP_iy and $yP_jx\}$.

Position Change: Reasoned Change

$d \in \mathcal{R}^m$	R_1	xP_2y	yP_3x	d^*
xP_1y	$R_{1,1}$	$xP_{1,2}y$	$yP_{1,3}x$	yP_1^*x
R_2	$R_{2,1}$	$R_{2,2}$	$R_{2,3}$	R_2^*
yP_3x	$R_{3,1}$	$R_{3,2}$	$R_{3,3}$	yP_3^*x

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3. For all $x, y \in X$, for all $i \in \{1, \ldots, m\}$, $[xP_iy \& yP_i^*x] \Rightarrow [$ there is no j such that $yP_jx \& xP_j^*y]$.

Results: Simple Majority Rule

Theorem

Let $X = 3$ and $m = 3$. If $F : \mathcal{R}^m \rightarrow \mathcal{D}^*, \mathcal{D}^* \subseteq \mathcal{R}^m$, satisfies Axiom 1, 2 and 3 then $\mathcal{D}^*$ satisfies Condition Value Overlap.

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Definition (Value Overlap)

Let $R_i \mid_{\{x,y,z\}}$ denote the restriction of binary relation R_i to the alternatives x, y and z. $\mathcal{D}^* \subseteq \mathcal{R}^m$ satisfies Value Overlap if, and only if,
$\mathcal{D}^* = \{d \in \mathcal{R}^m \mid$ for all $x, y, z \in X, \bigcap_{i=1}^{i=m} R_i \mid_{\{x,y,z\}} \neq \{(x,x),(y,y),(z,z)\}\}$.

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Theorem (Follows from Fishburn 1970)

If $\mathcal{D}^* \subseteq \mathcal{R}^m$ satisfies Value Overlap, then Simple Majority Rule yields a transitive social ranking.

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Theorem

If $\mathcal{D}^* \subseteq \mathcal{R}^m$ satisfies Value Overlap, then a Quota Rule generates an acyclic binary relation if,
(a) m is odd and $\frac{m+1}{2} \leq p$ or
(b) m is even and $\frac{m}{2} + 1 \leq p$.

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If $p = m$, Value Overlap restricts incompleteness.

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- (Social) Choice Framework allows for Specification of 'Action-Guidance'
- Lessons from Existing Results: Action-Guidance Limited!
- Extending the Framework:
  - Acyclicity Guaranteed for all $\frac{m+1}{2} \leq p \leq m$ (if m is odd) and $\frac{m}{2} + 1 \leq p \leq m$ (if m is even)
  - Incompleteness Restricted!

Open Questions & Future Research

- How Convincing is Completeness?

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$R*_1$	$R*_2$	$R*_3$
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'Reasoned Consensus' and 'Unreasoned Consensus'?
Solution: Introducing an External Perspective?

Thank You.