# Strategic Majoritarian Voting with Propositional Goals

Arianna Novaro

IRIT, University of Toulouse

Umberto Grandi  Dominique Longin  Emiliano Lorini

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B: "Suburbs restaurant and posters. No idea for proceedings."

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. . . we will use propositional logic.

# Talk Outline

1. Goal-based Voting
   Framework, Rules and Axioms

2. Strategic Behaviour
   Manipulation Strategies and Language Restrictions

3. Computational Complexity
   Winner Determination and Manipulation

4. Conclusions

# Goal-Based Voting

# Formal Framework

- $n$ agents in $\mathcal{N}$ have to decide over $m$ binary issues in $\mathcal{I}$
  - $\mathcal{N} = \{A, B, C\}$ and $\mathcal{I} = \{\texttt{proc}, \texttt{post}, \texttt{close}_{\texttt{rest}}\}$
- agent $i$ has for individual goal a propositional formula $\gamma_i$, whose models are in the set Mod$(\gamma_i)$
  - $\gamma_C = \texttt{proc} \wedge (\texttt{post} \rightarrow \texttt{close}_{\texttt{rest}})$
  - Mod$(\gamma_C) = \{(111), (101), (100)\}$
- a goal-profile $\mathbf{\Gamma} = (\gamma_1, \ldots, \gamma_n)$ contains all agents' goals
- no integrity constraints

# Goal-based Voting Rules

A goal-based voting rule is a collection of functions for all $n$ and $m$

$$F : (\mathcal{L}_{\mathcal{I}})^n \to \mathcal{P}(\{0,1\}^m) \setminus \{\emptyset\}$$

Approval: Return all interpretations satisfying the most goals.

Majority: ... how to generalize to propositional goals?

# Issue-wise Majority Rules

| Agent $i$ | $\gamma_i$ | $\text{Mod}(\gamma_i)$ |
|---|---|---|
| A | $\neg\texttt{proc} \wedge \neg\texttt{post} \wedge \neg\texttt{close}_{\texttt{rest}}$ | (000) |
| B | $\texttt{post} \wedge \neg\texttt{close}_{\texttt{rest}}$ | (010)<br>(110) |
| C | $\texttt{proc} \wedge (\texttt{post} \rightarrow \texttt{close}_{\texttt{rest}})$ | (111)<br>(101)<br>(100) |

- **EMaj** Majority with equal weights to models.
- **TrueMaj** Majority with equal weights to models and fair treatment of ties.
- **2sMaj** Majority done in two steps: on goals, and then on result of step one.

# Characterization of *TrueMaj*

Classical (and new) axioms defined for goal-based voting.

**Theorem.**
A rule is egalitarian, independent, neutral, anonymous, positive responsive, unanimous and dual if and only if it is *TrueMaj*.

# Strategic Behaviour

# What if Agents Lie?


A: "Proceedings, posters, close restaurant."
B: "No proceedings, posters, suburbs restaurant."
C: "Either *no proceedings, posters and close-by restaurant*, or *no posters and suburbs restaurant*."

| | | |
|---|---|---|
| A | (111) | (111) |
| B | (010) | (010) |
| C | (011)<br>(100)<br>(000) | (011) |
| *TrueMaj* | (010) | (011) |

# Two Notions of Resoluteness

$F$ is resolute if it always returns a singleton output.

- *EMaj* and *2sMaj* are resolute.

(!) Resoluteness incompatible with anonymity and duality.

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$F$ is weakly resolute if on all $\mathbf{\Gamma}$, $F(\mathbf{\Gamma}) = \text{Mod}(\varphi)$ for $\varphi$ a conjunction.

- Independence implies weak resoluteness.
- *TrueMaj* is weakly resolute.

# When are Agents Satisfied with Outcomes?

- $F$ is resolute: easy!
  An agent $i$ is satisfied with $F(\mathbf{\Gamma})$ *iff* $F(\mathbf{\Gamma}) \subset \text{Mod}(\gamma_i)$.

- $F$ is weakly resolute: it depends. . .

# When are Agents Satisfied with Outcomes?

- $F$ is resolute: easy!
  An agent $i$ is satisfied with $F(\mathbf{\Gamma})$ *iff* $F(\mathbf{\Gamma}) \subset \mathsf{Mod}(\gamma_i)$.

- $F$ is weakly resolute: it depends. . .
  - optimist: at least one of $i$'s goal models is in the outcome
  - pessimist: all models in outcome are models of $i$'s goal
  - expected utility maximizer: the more of $i$'s models in the outcome (wrt total models in outcome), the better

# Strategy-proofness

- Agent $i$'s preference on outcomes is:

$$F(\mathbf{\Gamma}) \succcurlyeq_i F(\mathbf{\Gamma}') \text{ \textit{iff} } \textit{sat}(i, F(\mathbf{\Gamma})) \geq \textit{sat}(i, F(\mathbf{\Gamma}')).$$

- Agent $i$ has an incentive to manipulate by submitting goal $\gamma_i'$ instead of $\gamma_i$ if and only if $F(\mathbf{\Gamma}_{-i}, \gamma_i') \succ_i F(\mathbf{\Gamma})$.

- A rule $F$ is strategy-proof if and only if for all profiles $\mathbf{\Gamma}$ there is no agent $i$ who has an incentive to manipulate.

# Manipulation Strategies and Results

Agents may know each other and have some ideas about their goals ...

Unrestricted: $i$ can send any $\gamma_i'$ instead of her truthful $\gamma_i$

Erosion: $i$ can only send a $\gamma_i'$ s.t. $\mathsf{Mod}(\gamma_i') \subseteq \mathsf{Mod}(\gamma_i)$

Dilatation: $i$ can send only a $\gamma_i'$ s.t. $\mathsf{Mod}(\gamma_i) \subseteq \mathsf{Mod}(\gamma_i')$

| | $\mathcal{L}$ | | $\mathcal{L}^{\wedge}$ | | $\mathcal{L}^{\vee}$ | | $\mathcal{L}^{\oplus}$ | |
|---|---|---|---|---|---|---|---|---|
| | E | D | E | D | E | D | E | D |
| *EMaj* | M | M | SP | SP | M | SP | M | M |
| *TrueMaj* | M | M | SP | SP | M | SP | M | M |
| *2sMaj* | M | M | SP | SP | SP | SP | M | M |

# Computational Complexity

# Majorities are (PP-)Hard

WinDet($F$): given profile and issue, issue is true in outcome?
Manip($F$): given profile and agent $i$, can agent $i$ manipulate?

WinDet(*2sMaj*) and WinDet(*EMaj*) are PP-hard.

Manip(*2sMaj*) and Manip(*EMaj*) are PP-hard.

# Conclusions

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- New framework for group decision-making: goal-based voting
  - Close to *Judgment Aggregation* (with/without abstentions) and to *Belief Merging*, but different
- Adaptation of voting rules in many ways (focus on *majorities*)
- Adaptation of axioms in many ways (e.g., *resoluteness*)
  - A characterization of *TrueMaj*
- A study of manipulation when agents behave strategically
  - Different *strategies* that agents are allowed to use
  - *Language restrictions* bring strategy-proofness
- Hard complexity results for WINDET and MANIP