Formal Languages in Logic
A cognitive perspective

Catarina Dutilh Novaes
Faculty of Philosophy
University of Groningen
Introduction

- What is the cognitive impact of using formal languages when doing logic?
- Historical observation: in logical traditions where no extensive use of formalisms was made, the output is significantly different.
- Formal languages/formalisms: a technology gradually developed to provide a ‘cognitive boost’.
- A chapter in the history of mathematical notations, which in turn is a chapter in the history of writing.
Introduction

- What is the cognitive impact of using formal languages when doing logic?
- Historical observation: in logical traditions where no extensive use of formalisms was made, the output is significantly different.
- Formal languages/formalisms: a technology gradually developed to provide a ‘cognitive boost’.
- A chapter in the history of mathematical notations, which in turn is a chapter in the history of writing.
Introduction

- What is the cognitive impact of using formal languages when doing logic?
- Historical observation: in logical traditions where no extensive use of formalisms was made, the output is significantly different.
- Formal languages/formalisms: a technology gradually developed to provide a ‘cognitive boost’.
- A chapter in the history of mathematical notations, which in turn is a chapter in the history of writing.
Introduction

- What is the cognitive impact of using formal languages when doing logic?
- Historical observation: in logical traditions where no extensive use of formalisms was made, the output is significantly different.
- Formal languages/formalisms: a technology gradually developed to provide a ‘cognitive boost’.
- A chapter in the history of mathematical notations, which in turn is a chapter in the history of writing.
Main claims in the book

- ‘Doing logic’ with formal languages: manipulating the notations (bodily engagement) is *constitutive* of the cognitive processes in question.
- Using formalisms helps us counterbalance some of our default reasoning tendencies, which are not necessarily advantageous in some contexts: *doxastic conservativeness*.
- Formal languages do so by *neutralizing semantic activation* and by *externalizing reasoning processes*.
Main claims in the book

- ‘Doing logic’ with formal languages: manipulating the notations (bodily engagement) is *constitutive* of the cognitive processes in question.
- Using formalisms helps us counterbalance some of our default reasoning tendencies, which are not necessarily advantageous in some contexts: *doxastic conservativeness*.
- Formal languages do so by *neutralizing semantic activation* and by *externalizing reasoning processes*.
Main claims in the book

• ‘Doing logic’ with formal languages: manipulating the notations (bodily engagement) is constitutive of the cognitive processes in question.
• Using formalisms helps us counterbalance some of our default reasoning tendencies, which are not necessarily advantageous in some contexts: doxastic conservativeness.
• Formal languages do so by neutralizing semantic activation and by externalizing reasoning processes.
Formalisms as merely auxiliary

• What is the impact of *perceptual features* of a formalism on reasoning processes?
• Orthodox view: none, as any ‘isomorphic’ formalism should be cognitively equivalent (Carnap).
• Primacy of ‘internal’, amodal operations.
Formalisms as merely auxiliary

- What is the impact of perceptual features of a formalism on reasoning processes?
- Orthodox view: none, as any ‘isomorphic’ formalism should be cognitively equivalent (Carnap).
- Primacy of ‘internal’, amodal operations.
Formalisms as merely auxiliary

- What is the impact of perceptual features of a formalism on reasoning processes?
- Orthodox view: none, as any ‘isomorphic’ formalism should be cognitively equivalent (Carnap).
- Primacy of ‘internal’, amodal operations.
Formalisms as external devices

- But when using formalisms, reasoning is largely (though not entirely) transferred to the *external realm*, in the form of rules of formation and transformation.

- Reasoning with formalisms is at least to some extent a process of ‘fiddling around’ with the symbols.
Formalisms as external devices

- But when using formalisms, reasoning is largely (though not entirely) transferred to the *external realm*, in the form of rules of formation and transformation.
- Reasoning with formalisms is at least to some extent a process of ‘fiddling around’ with the symbols.
Formalisms and ‘symbol-pushing’

- D. Landy: *perceptual manipulations theory*.
- “Symbolic reasoning is a special kind of embodied reasoning in which … logical formulae serve as the target of powerful, general perceptual and perceptual-motor systems.”
- Reasoning with formal languages is inherently *modal*.
  => The specific perceptual properties of external notations are crucial: recursive applications of bodily engagement with some system of notation.
- The agent literally ‘moves’ bits and pieces of the notation around to perform transformations.
Formalisms and ‘symbol-pushing’

• D. Landy: *perceptual manipulations theory.*

• “Symbolic reasoning is a special kind of embodied reasoning in which ... logical formulae serve as the target of powerful, general perceptual and perceptual-motor systems.”

• Reasoning with formal languages is inherently *modal.*

=> The specific perceptual properties of external notations are crucial: recursive applications of bodily engagement with some system of notation.

• The agent literally ‘moves’ bits and pieces of the notation around to perform transformations.
Formalisms and ‘symbol-pushing’

• D. Landy: perceptual manipulations theory:
• “Symbolic reasoning is a special kind of embodied reasoning in which … logical formulae serve as the target of powerful, general perceptual and perceptual-motor systems.”
• Reasoning with formal languages is inherently modal.
  => The specific perceptual properties of external notations are crucial: recursive applications of bodily engagement with some system of notation.
• The agent literally ‘moves’ bits and pieces of the notation around to perform transformations.
Formalisms and ‘symbol-pushing’

- D. Landy: *perceptual manipulations theory*.
- “Symbolic reasoning is a special kind of embodied reasoning in which … logical formulae serve as the target of powerful, general perceptual and perceptual-motor systems.”
- Reasoning with formal languages is inherently *modal*.
- The specific perceptual properties of external notations are crucial: recursive applications of bodily engagement with some system of notation.
- The agent literally ‘moves’ bits and pieces of the notation around to perform transformations.
Formalisms and ‘symbol-pushing’

- D. Landy: *perceptual manipulations theory*.
- “Symbolic reasoning is a special kind of embodied reasoning in which … logical formulae serve as the target of powerful, general perceptual and perceptual-motor systems.”
- Reasoning with formal languages is inherently *modal*.
  => The specific perceptual properties of external notations are crucial: recursive applications of bodily engagement with some system of notation.
- The agent literally ‘moves’ bits and pieces of the notation around to perform transformations.
Symbol-pushing

\[ a + b \times c = d \implies b \times c = d - a \]
Symbol-pushing

\[ a + b \times c = d \quad \Rightarrow \quad b \times c = d - a \]
Symbol-pushing

\[ \Gamma, A \Rightarrow B, \Delta \]

\[ \Gamma, A \Rightarrow B, A \rightarrow B, \Delta \]
Experimental results – spacing
(Landy & Goldstone 2007)

- Analysis of self-generated productions of handwritten arithmetic expressions and typewritten statements in logic.
- Conclusion: “substantial evidence for spatial representational schemes even in these highly symbolic domains.”
- Systematic introduction of spacing having no logical/formal function: spacing in handwritten equations reflects formal structure of equations, and hierarchy of connectives in logical formulae.
Experimental results – spacing
(Landy & Goldstone 2007)

- Analysis of self-generated productions of handwritten arithmetic expressions and typewritten statements in logic.

- Conclusion: “substantial evidence for spatial representational schemes even in these highly symbolic domains.”

- Systematic introduction of spacing having no logical/formal function: spacing in handwritten equations reflects formal structure of equations, and hierarchy of connectives in logical formulae.
Experimental results – spacing
(Landy & Goldstone 2007)

- Analysis of self-generated productions of handwritten arithmetic expressions and typewritten statements in logic.
- Conclusion: “substantial evidence for spatial representational schemes even in these highly symbolic domains.”
- Systematic introduction of spacing having no logical/formal function: spacing in handwritten equations reflects formal structure of equations, and hierarchy of connectives in logical formulae.
Experimental results – moving background
(Landy & Goldstone 2009)

• Task: to solve linear equations with one variable displayed against a background moving right or left.
• Goal: to probe the level of involvement of processes normally related to motion.
• If there is a ‘symbol pushing’ component, then direction of background movement should affect performance.
• Results: solving an equation was facilitated when the background motion moved in the direction of the symbolic transposition required.
Experimental results – moving background
(Landy & Goldstone 2009)

• Task: to solve linear equations with one variable displayed against a background moving right or left.
• Goal: to probe the level of involvement of processes normally related to motion.
• If there is a ‘symbol pushing’ component, then direction of background movement should affect performance.
• Results: solving an equation was facilitated when the background motion moved in the direction of the symbolic transposition required.
Experimental results – moving background
(Landy & Goldstone 2009)

- Task: to solve linear equations with one variable displayed against a background moving right or left.
- Goal: to probe the level of involvement of processes normally related to motion.
- If there is a ‘symbol pushing’ component, then direction of background movement should affect performance.
- Results: solving an equation was facilitated when the background motion moved in the direction of the symbolic transposition required.
Experimental results – moving background (Landy & Goldstone 2009)

- Task: to solve linear equations with one variable displayed against a background moving right or left.
- Goal: to probe the level of involvement of processes normally related to motion.
- If there is a ‘symbol pushing’ component, then direction of background movement should affect performance.
- Results: solving an equation was facilitated when the background motion moved in the direction of the symbolic transposition required.
Experimental results – moving background

\[ 1 + 2x = 4 \]
Experimental results – moving background

\[ 1 + 2x = 4 \]

Facilitation

\[ 1 + 2x = 4 - 1 \]
Experimental results – moving background

\[ 1 + 2x = 4 \]

Facilitation

\[ 1 + 2x = 4 - 1 \]

No facilitation

\[ 1 + 2x = 4 - 1 \]
Externalization of reasoning and computing

- To compute (calculate): to reason with no appeal to ‘insight or ingenuity’.
- Turing machine: mechanization and externalization of reasoning processes.
- Formalisms: externalization and activation of perceptual-motor systems – precisely what blocks the interference of ingenuity or insight.
- Blocking “presuppositions from sneaking in” (Frege).
Externalization of reasoning and computing

- To compute (calculate): to reason with no appeal to ‘insight or ingenuity’.
- Turing machine: mechanization and externalization of reasoning processes.
- Formalisms: externalization and activation of perceptual-motor systems – *precisely* what blocks the interference of ingenuity or insight.
- Blocking “presuppositions from sneaking in” (Frege).
Externalization of reasoning and computing

• To compute (calculate): to reason with no appeal to ‘insight or ingenuity’.
• Turing machine: mechanization and externalization of reasoning processes.

• Formalisms: externalization and activation of perceptual-motor systems – *precisely* what blocks the interference of ingenuity or insight.
• Blocking “presuppositions from sneaking in” (Frege).
Externalization of reasoning and computing

• To compute (calculate): to reason with no appeal to ‘insight or ingenuity’.
• Turing machine: mechanization and externalization of reasoning processes.

• Formalisms: externalization and activation of perceptual-motor systems – *precisely* what blocks the interference of ingenuity or insight.
• Blocking “presuppositions from sneaking in” (Frege).
“Great, a warehouse filled with miles and miles of rewritable tape! What are we ever going to do with this, Alan? …Alan?”

And thus the Turing Machine was born.
Whitehead on notations

By the aid of symbolism, we can make transitions in reasoning almost mechanically by the eye, which otherwise would call into play the higher faculties of the brain. It is a profoundly erroneous truism [...] that we should cultivate the habit of thinking what we are doing. The precise opposite is the case. Civilization advances by extending the number of important operations which we can perform without thinking about them. (Whitehead 1911, 61).