Radical inquisitive semantics*

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Contents

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1 Inquisitive semantics: propositions as proposals

Traditionally, the meaning of a sentence is identified with its *informative* content. In much recent work, this notion is given a dynamic twist, and the meaning of a sentence is taken to be its potential to change the 'common ground' of a conversation. The most basic way to formalize this idea is to think of the common ground as a set of possible worlds, and of a sentence as providing information by eliminating some of these possible worlds.

Of course, this picture is limited in several ways. First, when exchanging information sentences are not only used to provide information, but also—crucially—to *raise issues*, that is, to indicate which kind of information is desired. Second, the given picture does not take into account that updating the common ground is a *cooperative* process. One conversational participant cannot simply change the common ground all by herself. All she can do is *propose* a certain change. Other participants may react to such a proposal in several ways. In a cooperative conversation, changes of the common ground come about by mutual agreement.

In order to overcome these limitations, *inquisitive semantics* (Groenendijk, 2009; Mascarenhas, 2009; Groenendijk and Roelofsen, 2009; Ciardelli and Roelofsen, 2009, among others) starts with a different picture. It views propositions as proposals to update the common ground. Crucially, these proposals do not always specify just one way of updating the common ground. They may suggest alternative ways of doing so, among which the addressee is then invited to choose. Formally, a proposition consists of one or more *possibilities*. Each possibility is a set of possible worlds and embodies a possible way to update the common ground. If a proposition consists of two or more possibilities, it is *inquisitive*: it invites other participants to provide information in such a way that one or more of the proposed updates may be established. Inquisitive propositions raise an issue. They indicate which kind of information is desired. In this way, inquisitive semantics directly reflects the idea that information exchange consists in a cooperative dynamic process of raising and resolving issues.

A concrete implementation of inquisitive semantics for the language of propositional and first-order predicate logic has been specified in (Groenendijk and Roelofsen, 2009; Ciardelli and Roelofsen, 2009; Ciardelli, 2009a,b, among other places). Here we will argue that this system, to which we will refer as *conservative* inquisitive semantics, only partially captures the central underlying conception of sentences as expressing proposals to update the common ground. Subsequently, an enriched implementation will be presented and illustrated in some detail.

1.1 Logic and conversation

One of the advantages of a notion of meaning that captures both informative and inquisitive content is that it gives rise to a much wider spectrum of *logical relations* between sentences than a notion of meaning that captures only informative content. If meaning is identified with informative content, and propositions are taken to be sets of possible worlds, then there are just two logical relations that can be captured formally: *entailment* (set inclusion) and *consistency* (set overlap). If we adopt a notion of meaning that captures both informative and inquisitive content, and conceive of propositions as sets of possibilities, it becomes possible to characterize a much wider range of logical relationships between sentences.

The dominant focus of formal semantics on informative content is rooted in the historical fact that the logical tools that it uses were originally developed to reason about the validity of arguments. For this particular purpose, informative content is perhaps indeed the most crucial meaning component, and entailment and consistency are the central logical notions that need to be characterized.

However, there is no good reason why the analysis of language more generally should limit its notion of meaning to informative content, and its range of logical relations to entailment and consistency. In fact, other logical relations that one might like to capture readily come to mind. For instance, one might be interested in a formal characterization of when one sentence is entirely directed at *endorsing* the proposal expressed by another sentence, or when one sentence is entirely directed at *rejecting* the proposal raised by another sentence. To illustrate how inquisitive semantics facilitates such characterizations, let us zoom in on the first of these logical relations, which we refer to as *compliance*.

1.2 Compliance

Consider the sentence in (1), and the responses in (2a-b).

(1) Pete will play the piano or Mary will dance tonight.

(2) a. Yes, Pete will play the piano.
b. Yes, in fact Pete will play the piano and Mary will dance.
c. Yes, in fact Pete will play the piano and I ate spaghetti last night.

We take it that there is a pre-theoretical distinction between the responses in (2a-b) and the one in (2c). It is appropriate to say, even in the absence of any contextual information, that (2a-b) are entirely directed at endorsing the proposal expressed by (1). This is not true for (2c). In the absence of contextual information, we cannot conclude that (2c) is entirely directed at endorsing the proposal expressed by (1): the statement that I ate spaghetti last night can only be taken to serve this purpose in very specific contexts.

Can this pre-theoretical distinction be captured formally? Consider first a semantic theory whose notion of meaning only reflects informative content. In such a theory the meaning of a sentence is taken to be the set of possible worlds in which the sentence is true. Let's call this the *truth-set* of the sentence. It is easy to see that truth-sets are too coarse-grained to capture the distinction between (2a-b) and (2c): the truth-sets of (2a-c) are all strictly included in the truth-set of (1). There is no way to distinguish (2c) from the other responses on the basis of truth-sets alone.

In inquisitive semantics, where the meaning of a sentence reflects both informative and inquisitive content, the distinction *can* be captured. The sentence in (1) expresses a proposition consisting of two possibilities. One of these possibilities consists of all worlds in which Pete will play the piano, the other possibility consists of all worlds in which Mary will dance.

Now we could say that one sentence φ is a *compliant response* to another sentence ψ if and only if every possibility for φ either directly coincides with a possibility for ψ, or can be obtained from a set of possibilities for ψ by union and/or intersection.[1] This notion of compliance captures the distinction that is relevant here: the responses in (2a-b) count as compliant responses to (1), but (2c) does not.

1.3 Counter-compliance

Compliant responses are always of a *positive* nature: they *endorse* the given proposal, and—ideally—specify which of the proposed updates can be established. Of course, such responses play an important role in cooperative information exchange, and it is useful to have a logical characterization of their properties.

However, *negative* reponses, which *reject* a given proposal, and—ideally—specify why the proposed updates cannot be established in any way, are just as important in order to exchange information in an efficient, cooperative way.

In the development of inquisitive semantics so far, the focus has almost exclusively been on positive responses. And it turns out that the implementation presented in earlier work does indeed not provide the necessary tools to provide a parallel characterization of positive and negative responses. To illustrate this point, consider the sentence in (3), and the possible responses in (4):

(3) Pete will play the piano and Mary will dance tonight.

(4) a. No, Pete will not play the piano.
 b. No, Pete will not play the piano and Mary will not dance.
 c. No, Pete will not play the piano and I ate spaghetti last night.

This example is completely parallel to the one we saw above. Again, there is a pre-theoretical distinction between (4a-b) and (4c), which we would like to capture in formal terms. However, the current implementation of inquisitive semantics does not bring us very far in this case. The responses in (4a-c) all express a proposition

[1] In (Groenendijk and Roelofsen, 2009), we defined a slightly different notion of compliance. The difference will be discussed below, but does not affect the present argument.

consisting of a single possibility, and all these possibilities are disjoint from the unique possibility for (3). There is nothing that distinguishes (4a-b) from (4c). It is impossible to capture the distinction between these sentences just on the basis of the possibilities that they are assigned.

One way to treat positive and negative responses as equal citizens would be to formally respresent a proposal not just as a set of possibilities, but rather as a set of possibilities plus a set of *counter-possibilities*, where—roughly speaking—possibilities correspond to positive responses and counter-possibilities to negative responses. Once the notion of meaning is enriched in this way, we will not only be able to define a notion of compliance, but also a notion of *counter-compliance*, its negative counterpart. This approach will be explored below. We will see that it leads to a range of new perspectives, especially on the interpretation of questions and conditionals.

2 Radical inquisitive semantics

To keep things simple, we consider a propositional language here. The extension to the first-order case should be relatively straightforward.

Definition 1 (Language)**.** We consider a language whose formulas are built up from a finite set of proposition letters $\mathcal{P}$, using the operators $\vee$, $\wedge$, $\sim$, and $\rightarrow$. We will use $?\varphi$ as an abbreviation of $\varphi \vee \sim\varphi$.

The basic ingredients of the semantics are possible worlds and possibilities.

Definition 2 (Possible worlds and possibilities)**.** A *possible world* is a function from $\mathcal{P}$ to $\{0, 1\}$. A *possibility* is a set of possible worlds.

For any possibility α, $\overline{\alpha}$ will denote the *complement* of α, i.e., the set of all worlds not in α. For any formula φ, $|\varphi|$ will denote the possibility consisting of all worlds that make φ true in a classical setting. We will refer to $|\varphi|$ as the *truth-set* of φ.

Definition 3 below recursively defines, for every sentence φ in our language, the *proposition* $\lceil\varphi\rceil$ expressed by φ, and the *counter-proposition* $\lfloor\varphi\rfloor$ for φ. Both $\lceil\varphi\rceil$ and $\lfloor\varphi\rfloor$ will be sets of possibilities. We will refer to the elements of $\lceil\varphi\rceil$ as the *possibilities for* φ, and to the elements of $\lfloor\varphi\rfloor$ as the *counter-possibilities for* φ. The clauses of the definition will be illustrated right below.

Definition 3 (Radical inquisitive semantics)**.**

1. $\lceil p \rceil \coloneqq \{\, |p| \,\}$

   $\lfloor p \rfloor \coloneqq \{\, \overline{|p|} \,\}$

2. $\lceil \varphi \vee \psi \rceil \coloneqq \lceil \varphi \rceil \cup \lceil \psi \rceil$

   $\lfloor \varphi \vee \psi \rfloor \coloneqq \{\alpha \cap \beta \mid \alpha \in \lfloor \varphi \rfloor \text{ and } \beta \in \lfloor \psi \rfloor\}$

3. $\lceil \varphi \wedge \psi \rceil \coloneqq \{\alpha \cap \beta \mid \alpha \in \lceil \varphi \rceil \text{ and } \beta \in \lceil \psi \rceil\}$

   $\lfloor \varphi \wedge \psi \rfloor \coloneqq \lfloor \varphi \rfloor \cup \lfloor \psi \rfloor$

4. $\lceil {\sim}\varphi \rceil \coloneqq \lfloor \varphi \rfloor$

   $\lfloor {\sim}\varphi \rfloor \coloneqq \lceil \varphi \rceil$

5. $\lceil \varphi \rightarrow \psi \rceil \coloneqq \{\gamma_f \mid f \in \lceil \psi \rceil^{\lceil \varphi \rceil}\,\}$ where $\gamma_f \coloneqq \bigcap_{\alpha \in \lceil \varphi \rceil} \alpha \Rightarrow f(\alpha)$

   $\lfloor \varphi \rightarrow \psi \rfloor \coloneqq \{\alpha \Rightarrow \beta \mid \alpha \in \lceil \varphi \rceil \text{ and } \beta \in \lfloor \psi \rfloor\}$

The clause for implication is defined in terms of a two-place operator $\Rightarrow$, which remains to be specified. Notice that $\Rightarrow$ takes two possibilities as its input and yields a third possibility as its output. For concreteness and simplicity, we will define $\Rightarrow$ as material implication here. But in principle, any more sophisticated existing analysis of non-inquisitive conditionals could be ‘plugged in’ here. We will return to this point in section 4.

Definition 4 ($\Rightarrow$)**.** $\alpha \Rightarrow \beta \coloneqq \overline{\alpha} \cup \beta$.

The clauses in definition 3 specify, for each sentence of our formal language, what the possibilities and the counter-possibilities for that sentence are. Given these possibilities and counter-possibilities, we can define what it means for one sentence to be a (counter-)compliant response to another.

Definition 5 (Compliance and counter-compliance)**.**

- φ is a compliant response to ψ iff every possibility for φ either directly coincides with a possibility for ψ or can be obtained from a set of possibilities for ψ by applying union and/or intersection.

- φ is a counterpliant response to ψ iff every possibility for φ either directly coincides with a counter-possibility for ψ or can be obtained from a set of counter-possibilities for ψ by applying union and/or intersection.

The remainder of this section is entirely devoted to explaining and illustrating the semantics specified in definition 3. To keep the illustration as transparent as possible, we will mostly focus on compliant and counter-compliant responses that correspond one-to-one with the possibilities and counter-possibilities for the example sentences that we will be discussing. We will refer to these responses as *basic* compliant and counter-compliant responses. All other compliant and counter-compliant responses can be obtained from these basic ones by taking unions and intersections.

Definition 6 (Basic compliant and counter-compliant responses)**.**

- φ is a basic compliant response to ψ iff there is only one possibility for φ, and that possibility is also a possibility for ψ.
- φ is a basic counter-compliant response to ψ iff there is only one possibility for φ, and that possibility is also a counter-possibility for ψ.

Now let us turn to the clauses of definition 3. We start with the ones for atomic sentences, disjunction and conjunction.

2.1 Atoms, disjunction, and conjunction

Basic cases

Consider the following sentences, with their translations into our formal language:

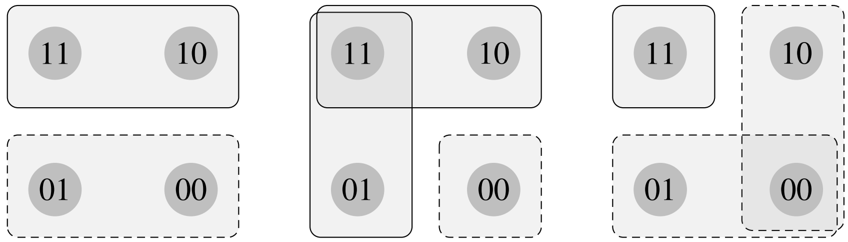
(5) Pete will play the piano. p

(6) Pete will play the piano or Sue will sing. $p \vee q$

(7) Pete will play the piano and Sue will sing. $p \wedge q$

Figure 1 depicts the possibilities and counter-possibilities for these sentences. In each sub-figure, there are just four possible worlds. In the world labeled 11, both p and q are true; in the world labeled 10, p is true but q is false, etcetera. Possibilities are depicted as rounded rectangles with solid borders, while counter-possibilities are depicted as rounded rectangles with dashed borders.

According to the atomic clause in definition 3, the proposition expressed by p is $\{\ |p|\ \}$, and the counter-proposition for p is $\{\ |{\sim}p|\ \}$. $|p|$ is the set of all worlds where p is true, which, in figure 1(a), is the set $\{11, 10\}$. Similarly, $|{\sim}p|$ is the set of all worlds where p is false, which, in figure 1(a), is the set $\{01, 00\}$.



(a) p (b) $p \vee q$ (c) $p \wedge q$

Figure 1: Possibilities and counter-possibilities for p, $p \vee q$, and $p \wedge q$.

These (counter-)possibilities correspond with the following (counter-)compliant responses:

(8) *Basic compliant response to (5):*
Yes, Pete will play the piano. p

(9) *Basic counter-compliant response to (5):*
No, Pete will not play the piano. $\sim p$

Now let us consider example (6). According to the clause for disjunction, $\lceil p \vee q \rceil = \{\, |p|, |q| \,\}$ and $\lfloor p \vee q \rfloor = \{\, |\sim p \wedge \sim q| \,\}$. These (counter-)possibilities, depicted in figure 1(b), correspond with the following (counter-)compliant responses:

(10) *Basic compliant responses to (6):*

a. Yes, Pete will play the piano. p
b. Yes, Sue will sing. q

(11) *Basic counter-compliant response to (6):*

a. No, Pete will not play the piano and Sue will not sing. $\sim p \wedge \sim q$

Finally, consider example (7). According to the clause for conjunction, $\lceil p \wedge q \rceil = \{\, |p \wedge q| \,\}$ and $\lfloor p \wedge q \rfloor = \{\, |\sim p|, |\sim q| \,\}$. These (counter-)possibilities, depicted in figure 1(c), correspond with following (counter-)compliant responses:

(12) *Basic compliant response to (7):*

a. Yes, Pete will play the piano and Sue will sing. $p \wedge q$

(13) *Basic counter-compliant responses to (7):*

a. No, Pete will not play the piano. $\sim p$
b. No, Sue will not sing. $\sim q$

Notice the symmetry between disjunction and conjunction. Disjunctive sentences like (6) are inquisitive, in the sense that the proposition they express consists of more than one possibility. As a consequence, they license more than one compliant response. Conjunctive sentences like (7) are *counter-inquisitive*, in the sense that they are associated with more than one counter-possibility and as such license more than one counter-compliant response.[2]

Interaction between disjunction and conjunction

Now let us turn to an example that illustrates the interaction between disjunction and conjunction:

(14) Pete will play the piano, and Sue will sing or Mary will dance. $p \wedge (q \vee r)$

(15) *Basic compliant responses:*

a. Yes, Pete will play the piano and Sue will sing. $p \wedge q$

b. Yes, Pete will play the piano and Mary will dance. $p \wedge r$

(16) *Basic counter-compliant responses:*

a. No, Pete will not play the piano. $\sim p$

b. No, Sue will not sing and Mary will not dance. $\sim q \wedge \sim r$

The clause for disjunction tells us that:

(17) $\lceil q \vee r \rceil = \lceil q \rceil \cup \lceil r \rceil = \{ |q|, |r| \}$

Now, the fact that conjunction involves *pointwise* intersection becomes crucial. In example (7) above this was not so essential, as both conjuncts were atomic, and expressed a proposition consisting of just one possibility. But in (14), the second conjunct is non-atomic, and expresses a proposition consisting of two possibilities.

(18) $\lceil p \wedge (q \vee r) \rceil = \{\alpha \cap \beta \mid \alpha \in \lceil p \rceil \text{ and } \beta \in \lceil q \vee r \rceil\} = \{ |p \wedge q|, |p \wedge r| \}$

So there are two possibilities for $p \wedge (q \vee r)$, each corresponding to one of the basic compliant responses in (15).[3]

Now let us turn to the counter-possibilities for $p \wedge (q \vee r)$. First, the clause for disjunction tells us that:

[2] Provide variants of these pictures when talking about negation, and about polar questions.

[3] As this example involves three atomic proposition letters, we would need three-dimensional graphics to visualize the relevant possibilities in a transparent way.

(19) $\lfloor q \vee r \rfloor = \{ \,|{\sim}q| \cap |{\sim}r|\, \} = \{ \,|{\sim}q \wedge {\sim}r|\, \}$

Next, the clause for conjunction yields:

(20) $\lfloor p \wedge (q \vee r) \rfloor = \lfloor p \rfloor \cup \lfloor q \vee r \rfloor = \{ \,|{\sim}p|\,,\ |{\sim}q \wedge {\sim}r|\, \}$

Thus, there are two counter-possibilities for $p \wedge (q \vee r)$, each corresponding to one of the basic counter-compliant responses in (16).

Some more complex (counter-)compliant responses

It is perhaps good to emphasize at this point that the two *basic* counter-compliant responses in (16) are not the only counter-compliant responses to (14), and certainly they are not the only sentences that could be thought of as rejecting the proposal expressed by (14). For instance, (21a) and (21b) reject that proposal just as much:

(21) a. No, either Pete won't play the piano,
or Sue will not sing and Mary will not dance. $\sim p \vee (\sim q \wedge \sim r)$

b. No, Pete will not play the piano,
Sue will not sing, and Mary will not dance. $\sim p \wedge \sim q \wedge \sim r$

Even though these responses do not directly correspond to any of the individual counter-possibilities for $p \wedge (q \vee r)$, there is still a rather straightforward indirect connection: the proposition expressed by (21a) consists of two possibilities, which are the two counter-possibilities for $p \wedge (q \vee r)$, and the proposition expressed by (21b) consists of one possibility, which is the *intersection* of the two counter-possibilities for $p \wedge (q \vee r)$. As such (21a) and (21b) do not count as *basic* counter-compliant responses to (14), but they do count as counter-compliant responses.

Similarly, (15a) and (15b) are not the only compliant responses to (14), and they are certainly not the only sentences that could reasonably be thought of as positive responses to (14). Other sentences that would naturally be classified as such include (22a) and (22b):

(22) a. Yes, Pete will play the piano and Sue will sing,
or Pete will play the piano and Mary will dance. $(p \wedge q) \vee (p \wedge r)$

b. Yes, in fact Pete will play the piano,
Sue will sing, and Mary will dance. $p \wedge q \wedge r$

Again, there is a straightforward connection with the possibilities for $p \wedge (q \vee r)$, even though there is no one-to-one correspondence: the proposition expressed by (22a) consists of two possibilities, which are the two possibilities for $p \wedge (q \vee r)$, and the proposition expressed by (22b) consists of one possibility, which is the intersection of the two possibilities for $p \wedge (q \vee r)$. As such (22a) and (22b) count as compliant responses to (14), be it not of the most basic variant.

To keep the illustration of our semantics as transparent as possible, we will in the following subsections mostly focus on basic (counter-)compliant responses, but it is important to keep in mind that these are never claimed to be the only (counter-)compliant responses.[4] This said, let us now return to definition 3 and zoom in on the clause for negation.

2.2 Negation

The proposition expressed by $\sim\varphi$ is the counter-proposition for φ, and vice versa, the counter-proposition for $\sim\varphi$ is the proposition expressed by φ itself. To see what this amounts to, first consider the following example, where negation scopes over a disjunction:

(23) Sue will not sing or dance. $\sim(p \vee q)$

(24) *Basic compliant response:*

a. That's right, she won't sing and she won't dance. $\sim p \wedge \sim q$

(25) *Basic counter-compliant responses:*

a. That's not right, she wíll sing. p

b. That's not right, she wíll dance. q

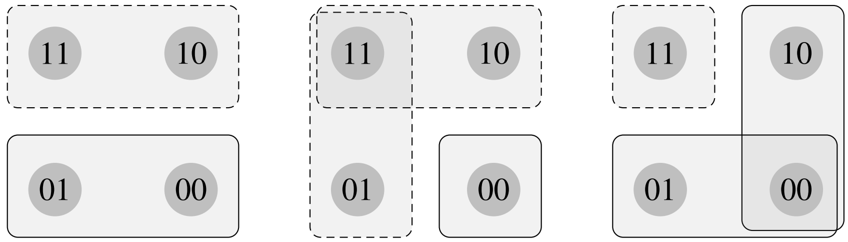
These responses correspond exactly with the possibilities and counter-possibilities for $\sim(p \vee q)$, which are visualized in figure 2(b).

(26) a. $\lceil \sim(p \vee q) \rceil = \lfloor p \vee q \rfloor = \{\, |\sim p \wedge \sim q| \,\}$

b. $\lfloor \sim(p \vee q) \rfloor = \lceil p \vee q \rceil = \{\, |p|,\ |q| \,\}$

There are also two more complex counter-compliant responses:

[4] We think that there is an interesting pragmatic difference between compliant responses that are obtained from basic ones by *intersection*, and ones that are not. Moreover, we think that this difference is typically marked by interjections like 'in fact', as in (22b). An account of this phenomenon will be provided in section 7.



(a) $\sim p$ (b) $\sim(p \vee q)$ (c) $\sim(p \wedge q)$

Figure 2: Possibilities and counter-possibilities for $\sim p$, $\sim(p \vee q)$, and $\sim(p \wedge q)$.

(27) a. That's not right, she wíll sing or dance. $p \vee q$
 b. That's not right, she wíll sing and she wíll dance. $p \wedge q$

The proposition expressed by (27a) consists of the two counter-possibilities for $\sim(p \vee q)$, and the proposition expressed by (27b) consists of the intersection of these two counter-possibilities.

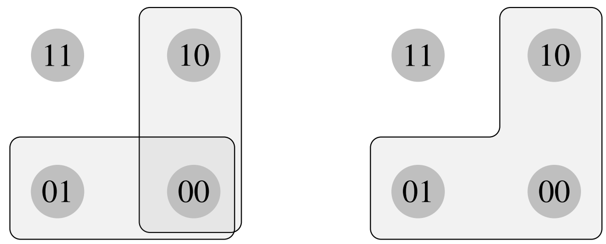
Next, consider an example in which negation scopes over a conjunction:

(28) Sue will not both sing and dance. $\sim(p \wedge q)$
 a. *Basic compliant responses:*
 That's right, she will not sing. $\sim p$
 That's right, she will not dance. $\sim q$
 b. *Basic counter-compliant response:*
 That's not right, she wíll both sing and dance. $p \wedge q$

Again, these responses correspond with the possibilities and counter-possibilities for $\sim(p \wedge q)$, which are depicted in figure 2(c).

(29) a. $\lceil \sim(p \wedge q) \rceil = \lfloor p \wedge q \rfloor = \{\, |\sim p|,\ |\sim q| \,\}$
 b. $\lfloor \sim(p \wedge q) \rfloor = \lceil p \wedge q \rceil = \{\, |p \wedge q| \,\}$

Notice in particular that there are *two* possibilities for $\sim(p \wedge q)$, which means that it is inquisitive. This is not the case in conservative inquisitive semantics, where negation is defined differently. In fact, as far as the 'positive' clauses in definition 3 are concerned, this is the only place where radical inquisitive semantics diverges from the more conservative system. Let us therefore take a moment to examine this difference in somewhat more detail.



(a) $\lceil {\sim}(p \wedge q) \rceil$ (b) $\lceil \neg(p \vee q) \rceil$

Figure 3: Radical versus conservative negation.

Radical versus conservative negation

We will use the symbol $\neg$ for negation as it is defined in conservative inquisitive semantics, and we will use the terms *conservative negation* and *radical negation* to talk about $\neg$ and ${\sim}$, respectively, whenever the two need to be distinguished explicitly.

In radical inquisitive semantics, $\lceil {\sim}\varphi \rceil$ is defined as $\lfloor \varphi \rfloor$, that is, in terms of the counter-proposition for φ. In conservative inquisitive semantics, counter-propositions are not part of the picture, so $\lceil \neg\varphi \rceil$ cannot be defined in terms of the counter-proposition for φ. Instead, it is defined in terms of the proposition expressed by φ. More concretely, $\lceil \neg\varphi \rceil$ is defined to consist of a single possibility, which is the complement of the union of all the possibilities for φ.

Definition 7 (Conservative negation)**.** $\lceil \neg\varphi \rceil \;:=\; \{\, \overline{\bigcup \lceil \varphi \rceil} \,\}$

This means, in particular, that $\neg\varphi$ is never inquisitive. Conservative negation blocks inquisitiveness. This is a key difference with radical negation, which manifests itself most clearly in cases where negation scopes over conjunction, as in example (28) above.

The contrast between ${\sim}(p \wedge q)$ and $\neg(p \wedge q)$ is visualized in figure 3. Notice that this figure only depicts the possibilities for both sentences, not the counter-possibilities. This is because the counter-possibilities for $\neg(p \wedge q)$ were never defined—counter-possibilities were not present in the conservative setting.

Also notice that the single possibility for $\neg(p \wedge q)$ is the union of the two possibilities for ${\sim}(p \wedge q)$. This is a particular instance of a more general fact:

Fact 8 (Radical and conservative negation)**.**
For any φ that does not involve implication: $\lceil \neg\varphi \rceil = \{ \bigcup \lceil {\sim}\varphi \rceil \}$.

Now we could ask whether negation in languages like English is to be analyzed in terms of radical negation or rather in terms of conservative negation. We submit that the radical analysis is in fact more appropriate. One important reason to do so is the direct correspondence between the two basic compliant responses in (28a) and the two possibilities for $\sim(p \wedge q)$. If negation in English were analyzed as conservative negation, then we would never be able to recognize the responses in (28a) as compliant responses to (28).

Another reason to analyze English negation in terms of $\sim$ rather than $\neg$ will be presented in section 4.4, once we have explained the clause for implication in some detail. Before we turn to implication, however, let us briefly illustrate the behavior of the question operator, which is defined here in terms of radical negation.

2.3 Questions

Recall that $?\varphi$ is defined as an abbreviation of $\varphi \vee \sim\varphi$. To illustrate what the consequences are of this definition, let us first consider the interpretation of an atomic polar question $?q$. By definition, $\lceil ?q \rceil = \lceil q \vee \sim q \rceil$. According to the clause for disjunction, $\lceil q \vee \sim q \rceil = \lceil q \rceil \cup \lceil \sim q \rceil$. By the clause for negation, we have that $\lceil q \rceil \cup \lceil \sim q \rceil = \lceil q \rceil \cup \lfloor q \rfloor$, which, by the atomic clause, amounts to $\{\, |q|,\ |\sim q|\, \}$. So:

(30) $\lceil ?q \rceil = \{\, |q|, |\sim q|\, \}$

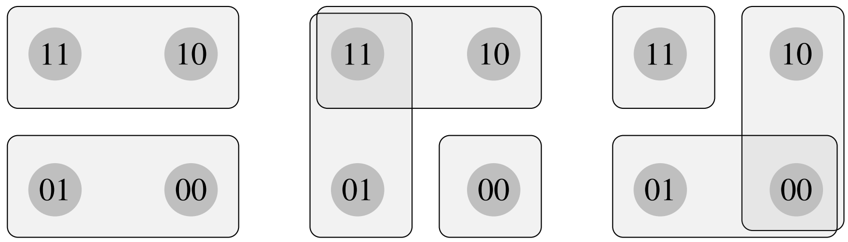
These possibilities are depicted in figure 4(a). The fact that $?q$ has these two possibilities means that it licenses two basic compliant responses, which, in languages like English, are the ones expressed by *yes* and *no*, respectively.

As for the counter-proposition expressed by $?q$, we have, by definition of $?q$ as an abbreviation of $q \vee \sim q$, that $\lfloor ?q \rfloor = \lfloor q \vee \sim q \rfloor$. By the counter-clause for disjunction, we have that $\lfloor q \vee \sim q \rfloor = \{\, \alpha \cap \beta \mid \alpha \in \lfloor q \rfloor \text{ and } \beta \in \lfloor \sim q \rfloor\, \}$, which reduces to $\{\, \alpha \cap \beta \mid \alpha \in \{\, |\sim q|\, \} \text{ and } \beta \in \{\, |q|\, \}\, \}$. So, what we end up with is:

(31) $\lfloor ?q \rfloor = \{\, \emptyset\, \}$

This means that there are no non-contradictory counter-compliant responses to $?q$. Thus, the analysis of atomic polar questions does not yield any surprises.

However, as questions are defined in terms of radical negation here, unlike in previous analyses, our general treatment of questions is bound to yield new predictions. In conservative inquisitive semantics, for instance, questions are defined in terms of conservative negation. That is, $?\varphi$ is defined as an abbreviation



(a) $\lceil ?p \rceil$ (b) $\lceil ?(p \vee q) \rceil$ (c) $\lceil ?(p \wedge q) \rceil$

Figure 4: Possibilities for atomic, disjunctive, and conjunctive polar questions. In each case, the only counter-possibility is $\emptyset$.

of $\varphi \vee \neg\varphi$ rather than $\varphi \vee {\sim}\varphi$. This happens to give exactly the same results for *atomic* polar questions. But the two theories start to diverge as soon as we go beyond the atomic case. Take, for instance, a simple conjunctive polar question:

(32) Will both Sue and Mary sing?

We translate (32) into our logical language as $?(q \wedge r)$. In radical inquisitive semantics, there are not just two, but three possibilities for this sentence, namely $|q \wedge r|$, $|{\sim}q|$, and $|{\sim}r|$. These possibilities, which are depicted in figure 4(c), correspond with the following basic compliant responses:

(33) *Basic compliant responses to* $?(q \wedge r)$ *in radical inquisitive semantics:*

a. Yes, both Sue and Mary will sing. $q \wedge r$
b. No, Sue won't sing. ${\sim}q$
c. No, Mary won't sing. ${\sim}r$

In conservative inquisitive semantics, the proposition expressed by $?(q \wedge r)$ consists of just two possibilities, $|q \wedge r|$ and $|{\sim}q \vee {\sim}r|$, corresponding to the following basic compliant responses:

(34) *Basic compliant responses to* $?(q \wedge r)$ *in conservative inquisitive sem:*

a. Yes, both Sue and Mary will sing. $q \wedge r$
b. No, either Sue or Mary won't sing. ${\sim}q \vee {\sim}r$

So, both systems generate the same *yes*-answer, but radical inquisitive semantics generates more specific *no*-answers than conservative inquisitive semantics does. Notice that (34b) counts as a compliant response in both systems, only in radical inquisitive semantics it does not count as a *basic* compliant response, since the proposition it expresses does not consist of a single possibility for $?(p \wedge q)$, but rather of two of these possibilities.

On the other hand, in conservative inquisitive semantics (33b) and (33c) cannot be classified as compliant responses in any straightforward way, because the proposition assigned to $?(p \wedge q)$ is too coarse-grained. This is clearly a shortcoming, which, for all we know, is shared by any other previous account of questions.

As a final illustration of how the question operator works, let us briefly consider a disjunctive polar question:

(35) Will either Sue or Mary sing?

We translate (35) as $?(q \vee r)$, which expresses exactly the same proposition in radical inquisitive semantics as it does in conservative inquisitive semantics.[5] As depicted in figure 4(b), this proposition consists of three possibilities, $|q|$, $|r|$, and $|\sim q \wedge \sim r|$, corresponding to the following compliant responses:

(36) a. Yes, Sue will sing.
b. Yes, Mary will sing.
c. No, neither Sue nor Mary will sing.

Notice that in the radical inquisitive setting there is a clear symmetry between conjunctive and disjunctive polar questions: in the case of conjunctive questions, the more fine-grained *no*-answers are made available, and in the case of disjunctive questions, the more fine-grained *yes*-answers are made available. Conservative inquisitive semantics, which only makes the more fine-grained *yes*-answers to disjunctive questions available, certainly makes some progress with respect to frameworks that do not license any of the more fine-grained answers, but this progress is taken a crucial step further in the radical setting.

There is much more to say about conjunctive and disjunctive polar questions, and other constructions like conjunctions and disjunctions of questions, and we will turn to that in section 7.[6] We now return to definition 3 and finally consider the clause for implication.

[5]This follows from the fact that $\sim(q \vee r)$ and $\neg(q \vee r)$ express the same proposition. As a consequence, $(q \vee r) \vee \sim(q \vee r)$ and $(q \vee r) \vee \neg(q \vee r)$ also express the same proposition.

[6]Depending on how much we get to say about this in section 7, we will need a footnote here saying that it is impossible to give a completely satisfactory analysis of disjunctive questions if we restrict ourselves to a purely propositional language. The interpretation of disjunctive questions is sensitive to intonation/subsentential focus structure, which can only be captured using a more expressive formal language (see Roelofsen and van Gool, 2009; Pruitt and Roelofsen, 2010).

2.4 Implication

The clause for implication is more involved than the ones for disjunction, conjunction, and negation. To illustrate it, we will consider four separate cases: one where the consequent of the implication is disjunctive, one where the antecedent is disjunctive, one where the consequent is conjunctive, and one where the antecedent is conjunctive.

Implication with a disjunctive consequent

The first example that we will use to illustrate the clause for implication is the conditional in (37). Notice that the consequent of this conditional is disjunctive (and therefore inquisitive). We want to derive that the basic compliant and counter-compliant responses to (37) are the ones specified in (37a) and (37b), respectively.

(37) If Pete plays the piano, then Sue will sing or Mary will dance.

a. *Basic compliant responses:*
Yes, if Pete plays the piano, Sue will sing.
Yes, if Pete plays the piano, Mary will dance.

b. *Basic counter-compliant response:*
No, if Pete plays the piano, Sue won't sing and Mary won't dance.

We translate (37) into our formal language as (38), and we will show that the proposition expressed by (38) and the counter-proposition for (38) are the ones in (38a) and (38b), respectively, which correspond exactly with the basic compliant and counter-compliant responses specified in (37a) and (37b).

(38) $p \to (q \vee r)$

(39) a. $\lceil p \to (q \vee r) \rceil = \{\, |p \to q|,\ |p \to r| \,\}$
b. $\lfloor p \to (q \vee r) \rfloor = \{\, |p \to (\sim q \wedge \sim r)| \,\}$

First consider the proposition $\lceil p \to (q \vee r) \rceil$. For convenience, let us repeat the clause for implication:

(40) $\lceil \varphi \to \psi \rceil \ :=\ \{\gamma_f \mid f\colon\ \lceil \varphi \rceil \to \lceil \psi \rceil\}$ where $\gamma_f \ :=\ \bigcap_{\alpha \in \lceil \varphi \rceil} \alpha \Rightarrow f(\alpha)$

The idea behind this clause is the following. The proposal expressed by a sentence can in general be *realized* in one or more ways. That is, if a proposal consists of just one possibility, then it proposes just one update, and it can be realized in exactly one way, namely by establishing that update. If a proposal consist of several possibilities, it proposes several possible updates, and this means that it can be realized in several ways, namely by establishing either one (or more) of the proposed updates. Incidentally, this is exactly what basic compliant responses do: they realize exactly one of the proposed updates.

Now, under this perspective, a conditional sentence $\varphi \to \psi$ can be thought of as expressing a proposal to establish a certain *implicational dependency* between the ways in which φ may be realized and the ways in which ψ may be realized, or, in more neutral terms, between the possibilities for φ and the possibilities for ψ. Such a dependency links every possibility $\alpha \in \lceil\varphi\rceil$ to some possibility $f(\alpha) \in \lceil\psi\rceil$, in such a way that for all $\alpha \in \lceil\varphi\rceil$, $\alpha \Rightarrow f(\alpha)$ holds.[7]

How many potential implicational dependencies there are depends on the number of possibilities for φ and ψ. If there are m possibilities for φ and n possibilities for ψ then there are n^m functions from $\lceil\varphi\rceil$ to $\lceil\psi\rceil$. Each of these functions f links every possibility $\alpha \in \lceil\varphi\rceil$ to some possibility $f(\alpha) \in \lceil\psi\rceil$. Thus, each of these functions corresponds with a potential implicational dependency between the possibilities for φ and the possibilities for ψ.[8]

In order to establish the implicational dependency corresponding to some function f from $\lceil\varphi\rceil$ to $\lceil\psi\rceil$, we have to establish that $\alpha \Rightarrow f(\alpha)$ holds for all $\alpha \in \lceil\varphi\rceil$. This means that we have to establish $\bigcap_{\alpha\in\lceil\varphi\rceil} \alpha \Rightarrow f(\alpha)$.[9] Notice that this intersection is a possibility, which is called γ_f in the clause for implication. For each function $f\colon \lceil\varphi\rceil \to \lceil\psi\rceil$, then, there is a corresponding possibility γ_f, and together, these possibilities make up the proposition expressed by $\varphi \to \psi$.

Now let us return to our example, $p \to (q \vee r)$. We have already seen that $\lceil p\rceil = \{\,|p|\,\}$ and $\lceil q\vee r\rceil = \{\,|q|, |r|\,\}$. Thus, there are two functions from $\lceil p\rceil$ to $\lceil q\vee r\rceil$, one that maps $|p|$ to $|q|$, and another one that maps $|p|$ to $|r|$. Call the first one f_q and the second one f_r. Then, the clause for implication tells us that $\lceil p \to (q \vee r)\rceil$ consists of two possibilities, γ_{f_q} and γ_{f_r}, where $\gamma_{f_q} = |p| \Rightarrow |q| = |p \to q|$ and

[7]Notice that the implicational dependencies in question do not necessarilly reveal any 'inherent dependency' between the possibilities for φ and the possibilities for ψ. In fact, the type of dependency that is involved here is completely determined by the interpretation of $\Rightarrow$. As mentioned above, there are many ways in which $\Rightarrow$ may be defined. If it is defined as material implication, as we did above, then the implicational dependency that $\varphi \to \varphi$ proposes to establish has nothing to do with any kind of inherent dependency. This is why we use the term 'implicational dependency'.

[8]In case the antecedent is not inquisitive, as in example (40), $\lceil\varphi\rceil$ contains just one possibility, and the number of functions from $\lceil\varphi\rceil$ to $\lceil\psi\rceil$ equals the number of possibilities in $\lceil\psi\rceil$.

[9]In case the antecedent is not inquisitive, as in example (40), $\lceil\varphi\rceil$ contains just one possibility α and $\bigcap_{\alpha\in\lceil\varphi\rceil} \alpha \Rightarrow f(\alpha)$ boils down to $\alpha \Rightarrow f(\alpha)$.

$\gamma_{f_r} = |p| \Rightarrow |r| = |p \rightarrow r|$. Thus, we obtain the desired result:

(41) $\lceil p \rightarrow (q \vee r) \rceil = \{\, |p \rightarrow q|,\ |p \rightarrow r|\, \}$

Next we turn to the counter-proposition for $p \rightarrow (q \vee r)$. Again, let us pause one moment to repeat the counter-clause for implication, and briefly explain the intuition behind it.

(42) $\lfloor \varphi \rightarrow \psi \rfloor \ := \ \{\alpha \Rightarrow \beta \mid \alpha \in \lceil \varphi \rceil \text{ and } \beta \in \lfloor \psi \rfloor\}$

As specified above, we think of $\varphi \rightarrow \psi$ as expressing a proposal to establish a certain implicational dependency between the possibilities for φ and the possibilities for ψ. Rejecting such a proposal, then, amounts to saying that none of the potential dependencies could possibly be established. This means that there must be some way of realizing φ that leads to the *rejection* of ψ. Thus, to reject $\varphi \rightarrow \psi$, we must point out that the realization of some possibility α for φ implies the realization of some counter-possibility β for ψ. This is why for every $\alpha \in \lceil \varphi \rceil$ and every $\beta \in \lfloor \psi \rfloor$, $\alpha \Rightarrow \beta$ is a counter-possibility for $\varphi \rightarrow \psi$.

There is always an upper bound for the number of counter-possibilities for $\varphi \rightarrow \psi$, which is determined by the number of possibilities for φ and the number of counter-possibilities for ψ: if there are m possibilities for φ and n counter-possibilities for ψ then there are at most $m \times n$ counter-possibilities for $\varphi \rightarrow \psi$.

Returning to our concrete example, the counter-possibilities for $p \rightarrow (q \vee r)$ are possibilities of the form $\alpha \Rightarrow \beta$, where $\alpha \in \lceil p \rceil$ and $\beta \in \lfloor q \vee r \rfloor$. Recall that $\lfloor q \vee r \rfloor = \{\, |{\sim}q \wedge {\sim}r|\, \}$. So, since there is only one possibility for the antecedent p and only one counter-possibility for the consequent $q \vee r$, there is also only one counter-possibility for the implication as a whole, which indeed corresponds with the counter-compliant response in (37b):

(43) $\lfloor p \rightarrow (q \vee r) \rfloor = \{\, |p \rightarrow ({\sim}q \wedge {\sim}r)|\, \}$

Implication with a disjunctive antecedent

If we reverse the antecedent and the consequent of example (37) we arrive at (44). Notice that the number of the basic compliant and counter-compliant responses is also reversed.

(44) If Sue sings or Mary dances, then Pete will play the piano.

a. *Basic compliant response:*
Yes, if Sue sings, Pete will play, and if Mary dances, he'll play too.

b. *Basic counter-compliant responses:*
No, if Sue sings Pete will not play.
No, if Mary dances Pete will not play.

We translate (44) as (45). The proposition expressed by (45) is (45a) and the counter-proposition for (45) is (45b), which correspond with the compliant and counter-compliant responses specified in (44a) and (44b).

(45) $(q \vee r) \rightarrow p$

a. $\lceil (q \vee r) \rightarrow p \rceil = \{\, |(q \rightarrow p) \wedge (r \rightarrow p)| \,\}$
b. $\lfloor (q \vee r) \rightarrow p \rfloor = \{\, |q \rightarrow {\sim}p|, |r \rightarrow {\sim}p| \,\}$

Since there is only a single possibility $|p|$ for the consequent of (45), there is only one function f that maps both possibilities $|q|$ and $|r|$ for the inquisitive antecedent of (45) to $|p|$. Thus, the single possibility γ_f for (45) is $(|q| \Rightarrow |p|) \cap (|r| \Rightarrow |p|)$, which, as (45a) reports, is the same as $|(q \rightarrow p) \wedge (r \rightarrow p)|$.

The counter-possibilities for (45) are of the form $\alpha \Rightarrow \beta$, where α is a possibility for the antecedent, $q \vee r$, and β is a counter-possibility for the consequent, p. The possibilities for $q \vee r$ are $|q|$ and $|r|$, and the only counter-possibility for p is $|{\sim}p|$. So there are two counter-possibilities for (45): $|q| \Rightarrow |{\sim}p|$ and $|r| \Rightarrow |{\sim}p|$, which, as (45b) reports, can also be written as $|q \rightarrow {\sim}p|$ and $|r \rightarrow {\sim}p|$.

Implication with a conjunctive antecedent

Next consider the conditional in (46), whose antecedent is conjunctive.

(46) If Sue will sing and Mary will dance, then Pete will play the piano.

(47) *Basic compliant response:*
Yes, if Sue will sing and Mary will dance, then Pete will play the piano.

(48) *Basic counter-compliant responses:*
No, if Sue will sing and Mary will dance, then Pete won't play the piano.

This case is simpler than the previous two: there is only one compliant and one counter-compliant response here. The translation of (48) into our formal language is given in (49). The proposition expressed by (49) is (49a) and the counter-proposition for (49) is (49b), which correspond with the compliant and counter-compliant responses specified in (48a) and (48b).

(49) $(q \wedge r) \rightarrow p$

a. $\lceil (q \wedge r) \rightarrow p \rceil = \{ \, |(q \wedge r) \rightarrow p| \, \}$
b. $\lfloor (q \wedge r) \rightarrow p \rfloor = \{ \, |(q \wedge r) \rightarrow {\sim}p| \, \}$

Notice that in this case there is only one possibility for the antecedent and one possibility for the consequent. So there is also only one function mapping possibilities for the antecedent to possibilities for the consequent. This is why $\lceil (q \wedge r) \rightarrow p \rceil$ consists of just one possibility.

Similarly, since there is only one counter-possibility for the consequent of the implication, and only one possibility for the antecedent, there is only one counter-possibility for the implication as a whole.

Implication with a conjunctive consequent

Now let us reverse the consequent and the antecedent of (46)

(50) If Pete plays the piano, then Sue will sing and Mary will dance.

(51) *Basic compliant response:*

a. Yes, if Pete plays the piano, then Sue will sing and Mary will dance.

(52) *Basic counter-compliant responses:*

a. No, If Pete plays the piano, then Sue won't sing.
b. No, If Pete plays the piano, then Mary won't dance.

This case is similar to the one where the antecedent was disjunctive: only one compliant response, but two basic counter-compliant responses. We translate (52) into our formal language as (53), which expresses the proposition in (53a) and has the counter-proposition in (53b). These correspond with the compliant and counter-compliant responses in (52a) and (52b).

(53) $p \rightarrow (q \wedge r)$

a. $\lceil p \rightarrow (q \wedge r) \rceil = \{ \, |p \rightarrow (q \wedge r)| \, \}$
b. $\lfloor p \rightarrow (q \wedge r) \rfloor = \{ \, |p \rightarrow {\sim}q|, \; |p \rightarrow {\sim}r| \, \}$

There is only one possibility for the antecedent and one possibility for the consequent in the case. Thus, there is also just one possibility for the implication as a whole. The fact that the implication has are two counter-possibilities stems from the fact that the consequent is conjunctive, and, as such, has two counter-possibilities.

This concludes our initial illustration of the radical inquisitive semantics specified in definition 3. The main aim of this initial illustration was to show, by means of examples, which possibilities and counter-possibilities the semantics assigns to the sentences of our formal language. It is perhaps good to emphasize that the semantics as such does not make any empirical predictions. It only provides the necessary tools to specify notions like entailment and compliance, which are supposed to correspond to certain pre-theoretical notions, and as such can be evaluated empirically.

As pointed out in the introduction, one important reason to move from a classical semantics to an inquisitive one is that the former only allows us to specify purely information-related notions such as entailment. In particular, it does not provide the necessary tools to specify notions such as compliance, which depend essentially on inquisitive content. Similarly, one important reason to move from a conservative implementation of inquisitive semantics to a more radical one is that the former allows us to define a notion of compliance, which generates predictions about positive responses, but not a notion of counter-compliance, which we would like to have in order to generate analogous predictions about negative responses.

Radical inquisitive semantics provides the necessary tools to define notions of compliance and counter-compliance. We provided such definitions, and illustrated the predictions that they generate for each of the examples that we discussed. At this point, however, we would like to emphasize two things: one is that our particular definition of compliant and counter-compliant responses is not forced upon us in any way by the proposed semantics. That is, the semantics in principle allows one to define many different notions of compliance and counter-compliance, and which of these definitions is 'right' is only determined by the empirical distinctions that one wants to capture, or more broadly, by whatever practical purpose one's logic is intended to serve.

The second point that we would like to emphasize is that compliance and counter-compliance are not necessarill the only notions that the semantics could be used to charaterize. These were the notions that it was specifically designed for, but once the machinery is in place, there could be other applications as well. Indeed, the next section starts to explore such additional applications.

3 Refusal, supposition and the Ramsey test

In order to characterize a broader range of possible responses to a given sentence, it is important to take into account not only the proposal that the sentence expresses, but also the more general issue that the sentence addresses. The form of sentences in natural language often indicates, at least partially, what this more general issue is. In particular, the underlying issue is often conveyed by the sentence's topic/focus structure, which, in a language like English, is typically reflected by intonation, by clefting, or by preposing certain constituents.

The formal language that we are working with here does not have enough expressive power to represent topic/focus structure. We could of course enrich our formal language, but, to keep things as transparent as possible, we will instead make the simple-minded assumption that the *question behind* a sentence φ is always the one expressed by $?\varphi$. Following common practice, we will sometimes also refer to this question as the *theme* of φ.

The ideas we will develop below are of course ultimately intended to be implemented based on a more sophisticated account of what determines the question behind a given sentence. But even under the simple-minded assumption that this question is always the one expressed by $?\varphi$, there will be some interesting things to say.

3.1 Refusing a proposal

A response to a given sentence does not always directly address the proposal expressed by that sentence itself. It may also address the question behind the given sentence. Thus, when analyzing the possible responses to a sentence φ, we should not only pay atttention to the compliant and counter-compliant responses to φ itself, but also to the compliant and counter-compliant responses to $?\varphi$.

Now, it turns out that there is a very particular relationship between the basic compliant responses to $?\varphi$ on the one hand, and the basic compliant and counter-complaint responses to φ on the other.

Fact 9 (Compliant responses to the question behind a sentence)**.**
A sentence ψ is a basic compliant response to $?\varphi$ if and only if ψ is a basic compliant or counter-complaint response to φ itself.

Proof. This follows immediately from the fact that $\lceil ?\varphi \rceil = \lceil \varphi \rceil \cup \lfloor \varphi \rfloor$. □

This is, in a sense, a negative result: it means that taking the basic compliant responses to $?\varphi$ into account will not allow us to understand a broader range of possible responses to φ. After all, every basic compliant response to $?\varphi$ can already be understood as a compliant or counter-compliant response to φ itself.

This is not the case for the *counter-compliant* responses to $?\varphi$. Taking these responses into account will genuinely allow us to understand a broader range of responses to φ. This is perhaps best illustrated by means of a concrete example. Consider the simple conditional in (54):

(54) If Pete will play the piano, Sue will sing. $p \to q$

The response that we are particularly interested in here is the one that *denies the antecedent* of the conditional:

(55) Pete will not play the piano. $\sim p$

This response does not count as a compliant or a counter-compliant response to the conditional itself. However, it does count as a counter-compliant response to the question behind the conditional:

(56) Will Sue sing if Pete will play the piano? $?(p \to q)$

a. *Basic compliant responses:*
Yes, Sue will sing if Pete will play the piano. $p \to q$
No, Sue won't sing if Pete will play the piano. $p \to \sim q$

b. *Basic counter-compliant response:*
Pete will not play the piano. $\sim p$

Notice that, in accordance with fact 9, the basic compliant responses to $?(p \to q)$ coincide with the basic compliant and counter-compliant responses to $p \to q$ itself. But the counter-compliant response to $?(p \to q)$ is a new element in the spectrum of possible responses.

With hindsight, the fact that the counter-compliant response to $?(p \to q)$ coincides with the denial of the antecedent of $p \to q$ makes perfect sense. If we think of a conditional as expressing a certain proposal, then to deny the antecedent of that conditional is not to endorse or reject that proposal, but to signal that the proposal cannot even be considered at all. And a sensible way to signal that the proposal expressed by a certain sentence cannot even be considered is to reject the question behind that sentence.

Definition 10 (Refusal)**.** If ψ is a counter-compliant response to $?\varphi$, then we will say that ψ *refuses the question behind* φ.[10]

[10] Notice that this is not an 'if and only if' statement. In section **??** we will give a complete characterization of refusals, which will also include sentences that are not counter-compliant responses to $?\varphi$.

In many cases $\lfloor ?\varphi \rfloor = \{\emptyset\}$, and there is no sensible way to refuse the question behind φ. In fact, this holds for all sentence in our formal language that do not involve implication:

Fact 11 (No refusal without implication)**.**
For any φ that does not involve implication: $\lfloor ?\varphi \rfloor = \{\emptyset\}$

However, as we have seen, the question behind a conditional sentence may well have non-absurd counter-possibilities, and as such may well be prone to refusal. In our particular example, $\sim p$ refuses the question behind $p \to q$.

3.2 Suppositions

Another way to conceive of $\sim p$ as a response to $p \to q$ is as a signal of *supposition failure*. It is natural to think of $p \to q$ as supposing that p is the case, and of the response $\sim p$ as going against this supposition. The present framework allows us to provide a formal characterization of the supposition of each sentence in our language. What is particularly interesting about this characterization is that it directly relates the notion of a supposition to the question behind a sentence.

The idea is that any sentence supposes that the question behind it cannot be refused. That is, a sentence φ always supposes that none of the counter-compliant responses to $?\varphi$ is true. Or yet in other words, φ supposes that the actual world is not included in any of the counter-possibilities for $?\varphi$. This is captured by the following definition.

Definition 12 (Supposition)**.** The *supposition* of φ is $\mathsf{sup}(\varphi) = \overline{\bigcup \lfloor ?\varphi \rfloor}$

Note that in case there are no non-absurd counter-possibilities for $?\varphi$, that is, if $\bigcup \lfloor ?\varphi \rfloor = \emptyset$, then the supposition of φ corresponds to ω, the set of all worlds, which means that the supposition of φ is trivial.

In case φ is itself a question $?\chi$, the question behind it is $??\chi$. In general the two are not fully equivalent. However, $\bigcup \lfloor ?\chi \rfloor$ will always coincide with $\bigcup \lfloor ??\chi \rfloor$. And this means that suppositions are always preserved under questions.

Fact 13 (Suppositions and questions)**.** For any φ: $\mathsf{sup}(\varphi) = \mathsf{sup}(?\varphi)$

Proof. The counter-propositions for $?\varphi$ and $??\varphi$ are as follows:

$$\begin{array}{rcl} \lfloor ?\varphi \rfloor & = & \{ \alpha \cap \beta \mid \alpha \in \lceil \varphi \rceil, \beta \in \lfloor \varphi \rfloor \} \\ \lfloor ??\varphi \rfloor & = & \{ \alpha \cap \beta \cap \gamma \mid \alpha \in \lceil \varphi \rceil, \beta \in \lfloor \varphi \rfloor, \gamma \in \lceil \varphi \rceil \cup \lfloor \varphi \rfloor \} \end{array}$$

It follows that $\bigcup \lfloor ?\varphi \rfloor = \bigcup \lfloor ??\varphi \rfloor$, which means that $\mathsf{sup}(\varphi) = \mathsf{sup}(?\varphi)$. □

Using this fact, we can also show that suppositions are preserved under radical negation.

Fact 14 (Supposition and negation)**.** For any φ: $\mathsf{sup}(\varphi) = \mathsf{sup}(\sim\varphi)$

Proof. First notice that for any φ, $?\varphi \equiv \varphi \vee \sim\varphi \equiv \sim\sim\varphi \vee \sim\varphi \equiv ?\sim\varphi$ (where $\equiv$ stands for equivalence: $\xi \equiv \psi$ means that $\lceil\xi\rceil = \lceil\psi\rceil$ and $\lfloor\xi\rfloor = \lfloor\psi\rfloor$). Now, according to fact 13, $\mathsf{sup}(\varphi) = \mathsf{sup}(?\varphi)$. The above equivalence yields that $\mathsf{sup}(?\varphi) = \mathsf{sup}(?\sim\varphi)$, and another application of fact 13 gives us that $\mathsf{sup}(?\sim\varphi) = \mathsf{sup}(\sim\varphi)$. Putting everything together yields the desired identity: $\mathsf{sup}(\varphi) = \mathsf{sup}(\sim\varphi)$. □

Being preserved under questions and negation is taken to be the hallmark of presuppositions. Facts 13 and 14 show that suppositions behave similar to presuppositions in this respect.

It should be noted that suppositions are preserved under radical negation, but not under the more conservative notion of negation. That is, $\neg\varphi$ and φ do not always have the same supposition. What is interesting, we think, is that the fact that suppositions are preserved under radical negation rests upon the more basic fact that suppositions are preserved under questions, which in turn ensues from the close formal connection that is established in the present framework between suppositions and the question behind a sentence.

3.3 Conditional questions

Consider the conditional question in (57):

(57) If Pete plays the piano, will Sue sing? $p \to ?q$

We translate (57) into our logical language as $p \to ?q$, which expresses the proposition specified in (58a) and has the counter-proposition in (58b):[11]

(58) a. $\lceil p \to ?q\rceil = \{\, |p \to q|,\ |p \to \sim q|\, \}$

b. $\lfloor p \to ?q\rfloor = \{\, |\sim p|\, \}$

As expected, the conditional question is inquisitive: $\lceil p \to ?q\rceil$ consists of two possibilities, which correspond to the two compliant responses in (59) below.

[11] Notice that $p \to ?q$ is equivalent with $p \to ?q$ in the present system. Thus, instead of treating (57) as a conditional question and translate it as $p \to ?q$, we could in principle just as well analyze it as a questioned conditional, and translate it as $?(p \to q)$. We will argue in section 3.5 that the former treatment is to be preferred.

More surprisingly, whereas we saw earlier that atomic questions do not license any sensible counter-compliant responses, we now see that conditional questions do: $\lfloor p \to ?q \rfloor$ contains a non-absurd possibility, which corresponds with the counter-compliant response in (60).

(59) *Basic compliant responses:*

a. Yes, if Pete plays the piano, then Sue will sing. $p \to q$

b. No, if Pete plays the piano, then Sue will not sing. $p \to {\sim}q$

(60) *Basic counter-compliant response:*

a. Pete will not play the piano. ${\sim}p$

This result offers a solution to a problem that many previous semantic theories of questions, including those of Hamblin (1973), Karttunen (1977), and Groenendijk and Stokhof (1984), had to leave unresolved. The problem is to provide a suitable characterization of responses to conditional questions that do not straightforwardly address the issue that such questions raise, but instead deny their antecedent. Isaacs and Rawlins (2008) provide a recent survey of the debate surrounding this problem, and a possible solution. The common intuition is that the relevant responses do not really resolve the given question, but that they do have, in the words of Isaacs and Rawlins, an *issue-dispelling* effect. This is perfectly in line with their treatment here as counter-compliant responses: as such, they do not resolve the given question, but rather reject the given proposal altogether. Clearly, this characterization crucially benefits from the fact that radical inquisitive semantics puts counter-compliant responses on an equal footing with compliant ones.

3.4 The Ramsey test

In the present framework, the conditional assertions in (61) and (62) contradict each other in a sense: (62) is a counter-compliant response to (61), i.e., it rejects the proposal expressed by (61), and vice versa, (61) rejects the proposal expressed by (62).

(61) If Pete plays the piano, then Sue will sing.

(62) If Pete plays the piano, then Sue will not sing.

This corresponds exactly to what Frank Ramsey wrote in his famous footnote in 1929, which is generally referred to as the *Ramsey test* for conditionals:

> If two people are arguing "If p will q?" and are both in doubt as to p, they are adding p hypothetically to their stock of knowledge and arguing on that basis about q; so that in a sense "If p, q" and "If p, $\sim q$" are contradictories.

In fact, in line with Ramsey's footnote, radical inquisitive semantics takes (61) and (62) to be the two opposing answers to the conditional question in (63):

(63) If Pete plays the piano, will Sue sing?

Moreover, the framework sheds new light on Ramsey's 'precondition', which says that the people who are arguing "If p will q?" are both in doubt as to p. In particular, it makes predictions about certain situations in which this precondition is *not* met, namely those situations in which one of the participants is in a position to *deny* p. In that case, (63) will be contested immediately with the counter-compliant response in (64), rejecting the conditional question and preventing other participants from 'hypothetically updating their stock of knowledge' with p.

(64) Pete will not play the piano.

So our semantics predicts that if two people are to argue, or more cooperatively, are to investigate "If p will q?", then neither of them should be in a position to deny p. If this precondition is satisfied they can hypothetically update their stock of knowledge with p, and investigate on that basis whether q.

Note that Ramsey's precondition also requires that neither of the participants is in a position to *affirm* p. It seems, however, that this part of the precondition is not quite necessary. The dialogue in (65) is perfectly felicitous, not showing any sign of non-cooperative behavior.

(65) A: If Pete will play the piano, will Sue then sing?
B: Yes, if Pete will play the piano, then Sue will sing.
A: Ah, then Pete will play the piano and Sue will sing.
B: Yes, indeed, Pete will play the piano and Sue will sing.

The conclusion that A draws from B's response shows that when asking the conditional question, A was in fact already in a position to affirm the antecedent. B's concluding remark shows that when answering the question A that started out with, she could have affirmed the antecedent, too.

So Ramsey's precondition of doubt can be weakened: we only need to assume that neither of the participants is in a position to deny the antecedent of the conditional question. And radical inquisitive semantics tells us why this assumption should be made: if one of the participants is in a position to deny the antecedent, he will not hypothetically update his stock of knowledge, but rather contest the question straightaway with a counter-compliant response.

3.5 Conditional questions and questioned conditionals

We saw above that the conditional question $p \to ?q$ is equivalent to the questioned conditional $?(p \to q)$. However, this is not a general fact about conditional questions and questioned conditionals. That is, $\varphi \to ?\psi$ is not always equivalent to $?(\varphi \to \psi)$. For instance, $(p \vee r) \to ?q$ is not equivalent to $?((p \vee r) \to q)$. These sentences express different propositions and counter-propositions:

(66) *Propositions:*

a. $$\lceil (p \vee r) \to ?q \rceil = \left\{ \begin{array}{c} |(p \to q) \wedge (r \to q)| \\ |(p \to \sim q) \wedge (r \to \sim q)| \\ |(p \to q) \wedge (r \to \sim q)| \\ |(p \to \sim q) \wedge (r \to q)| \end{array} \right\}$$

b. $$\lceil ?((p \vee r) \to q) \rceil = \left\{ \begin{array}{c} |(p \to q) \wedge (r \to q)| \\ |p \to \sim q| \\ |r \to \sim q| \end{array} \right\}$$

(67) *Counter-propositions:*

a. $\lfloor (p \vee r) \to ?q \rfloor = \{\, |\sim p|,\ |\sim r| \,\}$

b. $\lfloor ?((p \vee r) \to q) \rfloor = \{\, |\sim p \wedge (r \to q)|,\ |\sim r \wedge (p \to q)| \,\}$

This divergence raises two issues. One concerns the mapping between the sentences in our formal language and sentences in natural languages like English. In particular, one question we are lead to ask is whether English sentences like (68) should be analyzed as conditional questions or as questioned conditionals:

(68) If Pete or Sue plays the piano, will Mary dance?

In the present framework it is more appropriate to treat (68) as a conditional question, $(p \vee r) \to ?q$, than to treat it as a questioned conditional, $?((p \vee r) \to q)$. This becomes clear when considering the following two facts.

Fact 15. Every compliant response to $?((p \vee r) \to q)$ is also a compliant response to $(p \vee r) \to ?q$.

Proof. It is enough to show that every possibility for $?((p \vee r) \to q)$ either directly coincides with a possibility for $(p \vee r) \to ?q$, or can be obtained from a set of possibilities for $(p \vee r) \to ?q$ by applying union and intersection. In total, there are three possibilities for $?((p \vee r) \to q)$. The first one, $|(p \to q) \wedge (r \to q)|$, directly coincides with a possibility for $(p \vee r) \to ?q$. The second one, $|p \to {\sim}q|$, is the union of two possibilities for $(p \vee r) \to ?q$, namely $|(p \to {\sim}q) \wedge (r \to q)|$ and $|(p \to {\sim}q) \wedge (r \to {\sim}q)|$. And similarly, the third one, $|r \to {\sim}q|$, is also the union of two possibilities for $(p \vee r) \to ?q$, namely $|(r \to {\sim}q) \wedge (p \to q)|$ and $|(r \to {\sim}q) \wedge (p \to {\sim}q)|$. □

Fact 16. Not every compliant response to $(p \vee r) \to ?q$ is also a compliant response to $?((p \vee r) \to q)$.

Proof. For instance, $(p \to q) \wedge (r \to {\sim}q)$ is a compliant response to $(p \vee r) \to ?q$, but not to $?((p \vee r) \to q)$. □

These two facts show that $(p \vee r) \to ?q$ licenses strictly more compliant responses than $?((p \vee r) \to q)$. And these additional responses should indeed be licensed by a formal expression that is intended to correspond to the English sentence in (67). Consider, for instance, the response considered in the proof of fact 16: $(p \to q) \wedge (r \to {\sim}q)$. The English counterpart of this expression is:

(69) If Pete will play the piano, then Mary will dance,
but if Sue will play the piano, then Mary won't dance.

We certainly would like this sentence to count as a compliant response to (68). This is indeed achieved if (68) is analyzed as a conditional question, $(p \vee r) \to ?q$, but not if it is analyzed as a questioned conditional, $?((p \vee r) \to q)$. Thus, we conclude that the former analysis is more appropriate.

The other issue that arises is which of $(p \vee r) \to ?q$ and $?((p \vee r) \to q)$ should be conceived of as the question behind the conditional assertion $(p \vee r) \to q$. To resolve this issue we should in particular consider the counter-compliant responses to $(p \vee r) \to ?q$ and $?((p \vee r) \to q)$. These responses are supposed to signal supposition failure of $(p \vee r) \to q$, and this is indeed exactly what the counter-compliant responses to $(p \vee r) \to ?q$ do:

(70) *Basic counter-compliant responses to* $(p \vee r) \to ?q$:

a. Pete will not play the piano. $\quad {\sim}p$

b. Sue will not play the piano. $\quad {\sim}r$

The counter-compliant responses to $?((p \vee r) \to q)$ also signal supposition failure of $(p \vee r) \to q$, but they do *more* than that:

(71) *Basic counter-compliant responses to* $?((p \vee r) \to q)$*:*

a. Pete will not play the piano,
but if Sue will play, then Mary will dance. $\sim p \wedge (r \to q)$

b. Sue will not play the piano,
but if Pete will play, then Mary will dance. $\sim r \wedge (p \to q)$

Thus, $(p \vee r) \to ?q$ is more appropriate than $?((p \vee r) \to q)$ as a representation of the question behind $(p \vee r) \to q$. In fact, quite generally, $\varphi \to ?\psi$ is more appropriate than $?(\varphi \to \psi)$ as a representation of the question behind $\varphi \to \psi$.

This should be seen in the light of the qualifying remarks that we started out with in the beginning of this section: the working hypothesis that the question behind a sentence φ is always suitably represented by $?\varphi$ was deliberately simplistic. We mentioned that the role of topic/focus structure should ultimately be taken into account, and, arguably, *if*-clauses partly determine such topic/focus structure. Thus, it is not so surprising that a detailed examination of conditionals, even if confined by the limited expressive power of our formal language, confirms that our working hypothesis should be refined in certain ways.

However, as mentioned at the outset, it has not been our main purpose here to provide a detailed account of what determines the question behind a given sentence. Instead, the main point of this section was to show that by taking into account not only the proposal that a sentence expresses, but also the question behind that sentence, it becomes possible to understand a broader range of responses, specifically the ones that signal supposition failure. This, in turn, gave rise to an interesting connection between the supposition of a sentence and the question behind it, and to a formal interpretation of the Ramsey test for conditionals.

4 Minimal change semantics for conditionals

The clause for implication in our semantics rests on the semantic operator $\Rightarrow$. We defined $\Rightarrow$ as material implication (see definition 4), but we are certainly not commited to this definition. As mentioned above, any more sophisticated existing analysis of conditionals could in principle be ‘plugged in’ here. In this section, we will consider a range of phenomena that seem to require such a more sophisticated analysis. Initial motivation comes from *un*conditionals.

4.1 Unconditionals

Consider (72), in a sense the reverse of the conditional question in (57):[12]

(72) Whether Pete plays the piano or not, Sue will sing.

Sentences of this kind are referred to as *concessive conditionals*, or *unconditionals*. Rawlins (2008) argues that they are conditional sentences whose antecedent is an interrogative clause. This suggests translating (72) as $?p \rightarrow q$. In our system, this is equivalent with $(p \rightarrow q) \wedge (\sim p \rightarrow q)$. From this equivalence, it should be clear that we predict (72) to license the following compliant and counter-compliant responses:

(73) *Basic compliant response:*

a. Yes, if Pete plays the piano Sue will sing,
and if he doesn't play, she will sing too. $(p \rightarrow q) \wedge (\sim p \rightarrow q)$

(74) *Basic counter-compliant responses:*

a. No, if Pete plays the piano, Sue won't sing. $p \rightarrow \sim q$

b. No, if Pete doesn't play the piano, Sue won't sing. $\sim p \rightarrow \sim q$

In principle, these results are as desired. However, there is a subtlety to note here. As a result of the fact that we defined $\Rightarrow$ as material implication, the proposition expressed by $(p \rightarrow q) \wedge (\sim p \rightarrow q)$ is actually the same as the one expressed by q. In a classical setting, and in conservative inquisitive semantics, these two formulas are in fact completely equivalent. That is not the case here, because the two formulas are assigned different counter-propositions. But we do predict that (72) expresses exactly the same proposition as (75), and this is clearly problematic.

(75) Sue will sing.

However, identifying the source of the problem, and fixing it in the obvious way, leads to a promising analysis. The source of the problem is that $\Rightarrow$ is treated as material implication, and the obvious fix is to redefine it along the lines of a more sophisticated existing analysis of conditionals. Suppose for instance, that we make the standard assumption, originating in the work of Stalnaker (1968) and Lewis (1973), that $\Rightarrow$ is sensitive to a *similarity order* between worlds:

[12] The benefit of using inquisitive semantics to analyze this type of sentences was pointed out to us by Stefan Kaufmann, and the particular analysis to be presented below follows, in essence, his insight. See Kaufmann (2009) for a slightly different account, largely in the same spirit.

(76) $\alpha \Rightarrow \beta \ := \ \{w \mid \textsc{min}_w(\alpha) \subseteq \beta\}$

where $\textsc{min}_w(\alpha)$ is the set of worlds that belong to α and do not differ more from w than any other world in α.

Under this assumption, $\lceil q \rceil$ and $\lceil ?p \rightarrow q \rceil$ differ in exactly the right way. The former still consists of a single possibility containing all worlds where q holds. $\lceil ?p \rightarrow q \rceil$, however, becomes stronger:

(77) $\lceil ?p \rightarrow q \rceil = \{ \gamma \}$

where $\gamma = \{w \mid \textsc{min}_w|p| \subseteq |q| \text{ and } \textsc{min}_w|{\sim}p| \subseteq |q| \}$

To see whether w belongs to γ we should not just check whether q holds at w, but rather we should look at all p-worlds that minimally differ from w and all ${\sim}p$-worlds that minimally differ from w, and check whether q holds in all those worlds. In the terms of our original natural language example, we should not just check whether Sue sings at w, but we should look at all worlds minimally different from w where Pete plays the piano, and at all worlds minimally different from w where Pete doesn't play the piano, and check whether Sue sings in all those worlds. This indeed appears to be a satisfactory analysis of (72).

The approach also seems to lead to a suitable account of other types of unconditionals, such as:

(78) Whoever plays the piano, Sue will sing.

Spelling out such an account, however, would require us to move to a first-order setting. This is left for another occasion.

4.2 Dependency

Unconditionals can function as counter-compliant responses to 'dependency statements'. Consider the following example:

(79) Whether Sue will sing depends on whether Pete will play the piano.

(80) *Basic compliant responses:*

a. Yes, Sue will sing if and only if Pete will play the piano.
b. Yes, Sue will sing if and only if Pete will not play the piano.

(81) *Basic counter-compliant responses:*

a. No, whether Pete will play the piano or not, Sue will sing.

b. No, whether Pete will play the piano or not, Sue will not sing.

'Whether Pete plays the piano' and 'whether Sue sings' are translated into our logical language as $?p$ and $?q$, respectively. To say that $?q$ depends on $?p$ is to say that p implies q and $\sim p$ implies $\sim q$, or vice versa, that p implies $\sim q$ and $\sim p$ implies q. Thus, (79) as a whole is translated as a disjunction $\delta \coloneqq \delta_1 \vee \delta_2$, where:

(82) $\delta_1 \coloneqq (p \to q) \wedge (\sim p \to \sim q)$

$\delta_2 \coloneqq (p \to \sim q) \wedge (\sim p \to q)$

There is a unique possibility for δ_1 and a unique possibility for δ_2, and together these two possibilities constitute the proposition expressed by δ:

(83) $$\lceil \delta \rceil = \left\{ \begin{array}{c} |(p \to q) \wedge (\sim p \to \sim q)| \\ |(p \to \sim q) \wedge (\sim p \to q)| \end{array} \right\}$$

The elements of $\lceil \delta \rceil$ correspond exactly with the compliant responses in (80). To compute the counter-possibilities for δ we first have to compute the counter-possibilities for δ_1 and for δ_2, and then take pairwise intersections. This gives us:

(84) $$\lfloor \delta \rfloor = \left\{ \begin{array}{c} |(p \to q) \wedge (\sim p \to q)| \\ |(p \to \sim q) \wedge (\sim p \to \sim q)| \\ \emptyset \end{array} \right\}$$

The first two counter-possibilities correspond to the two unconditional counter-compliant responses in (81); the third counter-possibility, $\emptyset$, does not correspond to any sensical response and can therefore be ignored.

Now consider the question behind (79):

(85) Does whether Sue will sing depend on whether Pete will play the piano?

There are four basic compliant responses to (85): the two that count as basic compliant responses to (79) and the two that count as basic counter-compliant responses to (79). There are no sensical counter-compliant responses to (85). This is exactly what our semantics predicts:

(86) a. $$\lceil ?\delta \rceil = \left\{ \begin{array}{ccc} |(p \to q) & \wedge & (\sim p \to q)| \\ |(p \to \sim q) & \wedge & (\sim p \to \sim q)| \\ |(p \to q) & \wedge & (\sim p \to \sim q)| \\ |(p \to \sim q) & \wedge & (\sim p \to q)| \end{array} \right\}$$

b. $\lfloor ?\delta \rfloor = \{ \emptyset \}$

4.3 Minimal change with disjunctive antecedents

As soon as we make $\Rightarrow$ sensitive to a similarity order between worlds, the proposed semantics provides a straightforward solution to an old puzzle concerning conditionals with disjunctive antecedent. The solution is very closely related to the one proposed by Alonso-Ovalle (2006). Let us first present the puzzle and the solution that radical inquisitive semantics has to offer. After that, we will consider the connection with Alonso-Ovalle's account.

Consider the following situation. Chris is playing a game in which he has to throw two coins onto a table. He wins the game if and only if both coins come up heads. The first coin that Chris has to throw is made of nickel, the second is made of silver. Chris has managed to partly sabotage the game: he has magnetized the first coin, and has placed a magnet under the table, in such a way that this first coin is guaranteed to come up heads. He couldn't do the same trick with the second coin, as that one is made of silver and therefore could not be magnetized.

In this context, consider the following conditional, whose antecedent is disjunctive:

(87) If Chris throws tails with the first coin or heads with the second, he wins. $(p \vee q) \rightarrow r$

Intuitively, this sentence is false in the given context. Certainly, if Chris throws tails with the first coin, he loses.

A standard minimal change semantics for conditionals would erroneously predict that (87) is true in the given context. For, according to such a semantics, we have to look at all the worlds that make the antecedent, $p \vee q$, true, and that differ minimally from the actual world. Such worlds are all ones in which the second coin came up heads, but not ones in which the first came up tails (because these are worlds where Chris did not sabotage the game, and such worlds do not differ minimally from the actual world). Thus, in all the relevant worlds q is true, and p is false, which means that Chris threw heads with both coins. In all these worlds Chris won the game, and therefore the conditional is predicted to be true.

Exactly the same problem arises for *counterfactual* conditionals (and this is in fact how the problem is usually presented). Suppose that in the above context Chris threw his coins, and the first came up heads, but the second came up tails.

Chris lost the game. Now consider the following counterfactual conditional:

(88) If Chris had thrown tails with the first coin or heads with the second, he would have won. $(p \vee q) \to r$

Again, intuitively this sentence is false in the given context. Certainly, if Chris had thrown tails with the first coin (which could only have happened if he had refrained from sabotaging the game), he would not have won.

A standard minimal change semantics for counterfactuals erroneously predicts (88) to be true in the given context. It looks at all the worlds that make the antecedent, $p \vee q$, true, and that differ minimally from the actual world. Such worlds are all ones in which the second coin came up heads, but not ones in which the first came up tails (because these are worlds where Chris did not sabotage the game, and such worlds do not differ minimally from the actual world). Thus, all the relevant worlds make q true, but p false, which means that Chris threw heads with both coins. In all these worlds Chris won the game, and therefore the counterfactual is predicted to be true.

In radical inquisitive semantics this prediction does not arise. That is, any world w that fits the given description is not contained in the unique possibility for $(p \vee q) \to r$. This immediately follows from the fact that, in radical inquisitive semantics, $(p \vee q) \to r$ is equivalent to $(p \to r) \wedge (q \to r)$. This means that, to check whether w is contained in the unique possibility for $(p \vee q) \to r$, we have to consider all p-worlds that differ minimally from w, and all q-worlds that differ minimally from w. In all these worlds, r must hold. But this is not the case, since all the relevant p-worlds are evidently ones where p holds (the first coin came up tails), and therefore r does not hold (Chris did not win the game).

As mentioned above, the way in which the problem is solved here, or rather the reason why it does not arise at all, is closely related to the account proposed by Alonso-Ovalle (2006). More specifically, it is related to one essential ingredient of that account, which is the so-called *alternative semantics* for disjunction that several other authors have recently argued for (Simons, 2005; Alonso-Ovalle, 2006; Aloni, 2007, among others).[13] The key feature of this semantics, as foretold by its name, is that disjunction 'introduces alternatives'. That is, the semantic value of a disjunctive constituent, [A or B], is a *set* consisting of the semantic value of A and the semantic value of B. In particular, if A and B are sentences, and $|A|$ and $|B|$

[13]The other ingredients of Alonso-Ovalle's account are certain specific assumptions about the syntax and semantics of counterfactual conditionals. Our account is, for all we know, compatible with these assumptions, but does not require them.

are the classical propositions that they express, then the semantic value assigned to [A or B] in alternative semantics is { $|A|$, $|B|$ }.

In a nutshell, Alonso-Ovalle's proposal concerning counterfactual conditionals with disjunctive antecents is that (i) the disjunctive antecedent introduces two alternatives, and (ii) to evaluate the counterfactual as a whole, we have to look at minimally different worlds where the first alternative holds, but also at minimally different worlds where the second alternative holds. Under these assumptions, (87) is indeed predicted to be false in the given context, as desired.

In inquisitive semantics, the proposition expressed by $p \vee q$ is also a set, { $|p|$, $|q|$ }, and in some sense, $|p|$ and $|q|$ are also thought of as 'alternatives' in this framework: they represent alternative updates and correspond with alternative compliant responses. However, the primary *reason* to treat disjunction as introducing alternatives is very different in the two frameworks. In alternative semantics, the main concern is to facilitate the semantic composition process, and to provide a suitable basis for Gricean pragmatic reasoning,[14] while in inquisitive semantics, the main concern is to define a notion of semantic meaning that captures both informative and inquisitive content, and the main reason to treat disjunction as introducing alternatives is the observation that disjunctive sentences can be used inquisitively. The fact that these independent concerns have led to the same conclusion concerning the formal treatment of disjunction is quite remarkable. However, it should be emphasized here that in the case of inquisitive semantics the suggested treatment of disjunction is certainly not the main point, but rather part of a much broader reconsideration of the notion of semantic meaning.

Related to this difference in underlying motivation, there is also a more concrete difference between the treatment of disjunction in the two frameworks. In alternative semantics, the overall notion of semantic meaning does not change: sentences are still taken to express classical propositions, sets of possible worlds. Therefore, the alternatives generated by disjunction have to be 'closed off' at some point in the semantic composition process, in order to obtain such a classical proposition. As a consequence, alternative semantics relies on the existence of a silent *existential closure* operator, which takes as its input a set of alternatives, and yields as its output the union of all these alternatives, which is a classical proposition. So, the alternatives generated by disjunction play a role 'locally', they affect the process of semantic composition, but they never figure explicitly in the overall meaning of a sentence.

[14]This second concern has not been illustrated here. See, for instance, Alonso-Ovalle (2006), chapters 3 and 4.

In inquisitive semantics, the alternative possibilities generated by disjunction do not have to be closed off at any point, because propositions in general are taken to be sets of possibilities. Thus, the alternatives may not only affect the composition process, they may also figure in the overall meaning of sentences, capturing, in particular, their inquisitive content.[15]

Finally, there is one way in which *radical* inquisitive semantics differs both from alternative semantics and from conservative inquisitive semantics. Namely, in the latter two frameworks, alternatives are *only* generated by disjunction (and by existential quantification in a first-order inquisitive setting). In radical inquistive semantics, alternatives can be generated in other ways as well. For instance, the proposition expressed by $\sim(p \wedge q)$ is $\{\,|{\sim}p|,\ |{\sim}q|\,\}$, just the same as that expressed by ${\sim}p \vee {\sim}q$. In the next subsection we will see that this dissociation between alternatives and disjunction is in fact needed to obtain a satisfactory treatment of conditionals: the 'problem' with disjunctive antecedents is in fact a more general problem for semantic theories whose notion of meaning is too coarse-grained.

4.4 Alternatives without disjunction

Consider the context specified above, and suppose, as before, that Chris' first coin came up heads, but his second coin came up tails, which means that he lost the game. Now consider the following counterfactual conditional:

(89) If Chris hadn't thrown heads with the first coin and tails with the second, he would have won.

Intuitively, this sentence is false. If Chris had thrown tails with both coins, for instance, he would certainly not have won.

A standard minimal change semantics for counterfactual conditionals, again, makes the wrong prediction here. For suppose that (89) is formally respresented as $\sim(p \wedge q) \rightarrow r$. Then according to a standard minimal change semantics, we have to look at minimally different worlds that make the antecedent, $\sim(p \wedge q)$, true. Evidently, those are worlds that make $p \wedge q$ false, and among these, the ones that differ minimally from the actual worlds are ones that make q false, but keep p

[15]There are good reasons why existential closure (or, in our terms, *non-inquisitive closure*) should be available in the system as an operator that *optionally* applies at certain points in the sub-sentential composition process. This is essentially to make sure that disjunction is not always forced to take wide scope over any other logical operator. But in inquisitive semantics it is not necessary to assume that a disjunction can *only* occur in the scope of existential closure.

true. In those worlds, Chris throws heads with both coins, and consequently wins the game. Thus, the counterfactual is predicted to be true.

In radical inquisitive semantics, the antecedent of (89) is translated as $\sim(p \wedge q)$ (recall that we argued in section (29) that this is a more appropriate translation than $\sim(p \wedge q)$ in the radical inquisitive setting). Thus, the counterfactual as a whole is translated as $\sim(p \wedge q) \rightarrow r$. Now, any world w that is compatible with the described situation is not contained in the unique possibility for $\sim(p \wedge q) \rightarrow r$. This immediately follows from the fact that $\sim(p \wedge q) \rightarrow r$ is equivalent with $(\sim p \rightarrow r) \wedge (\sim q \rightarrow r)$, which in turn is equivalent with $(\sim p \rightarrow r) \wedge (\sim q \rightarrow r)$. This means that, to check whether w is contained in the unique possibility for $\sim(p \wedge q) \rightarrow r$, we have to consider all $\sim p$-worlds that differ minimally from w, and all $\sim q$-worlds that differ minimally from w. In all these worlds, r must hold. But this is not the case, since all the relevant $\sim p$-worlds are evidently ones where p does not hold (the first coin came up tails), and therefore r does not hold either (Chris did not win the game).

Thus, the problem described in section 4.3, which is usually assumed to be charateristic for conditionals with disjunctive antecedents, is in fact a more general problem, which is avoided altogether in radical inquisitive semantics.

4.5 Strengthening versus simplification

In the literature on conditionals there has always been a tension between two particular rules of inference, *strengthening the antecedent* (STRENGTHENING) and *simplification of disjunctive antecedents* (SIMPLIFICATION).[16] The first says that strengthening the antecedent of a conditional is truth-preserving. For instance, it allows us to infer from $q \rightarrow r$ that $(p \wedge q) \rightarrow r$, for any p. It is widely agreed, based on examples like (90) and (91), that this should *not* be a valid inference rule.

(90) If Mary has an essay to write, she will study late in the library.
$\not\rightsquigarrow$ If the library is closed and Mary has an essay to write, she will study late in the library.

(91) If you strike this match, it will light.
$\not\rightsquigarrow$ If you stand in a storm and strike this match, it will light.

Indeed, the fact that material and strict implication validate STRENGTHENING is often presented as a basic argument against them, and in favor of a minimal change

[16]We thank Frank Veltman for bringing this to our attention.

semantics of the type we have seen above, which does not validate STRENGTHENING.

SIMPLIFICATION is a weaker inference rule: it does not make a claim about strengthening the antecedent of a conditional in general, but is concerned with one particular way of doing so: replacing a disjunctive antecedent with either one of the individual disjuncts. For instance, it allows us to infer from $(p \vee q) \rightarrow r$ that $p \rightarrow r$ and that $q \rightarrow r$. And this *is* widely agreed to be a valid inference pattern.[17] A standard minimal change semantics, however, does not validate SIMPLIFICATION. This is precisely the source of the 'problem' that we discussed in section 4.3. Thus, there is a dilemma: one important feature of minimal change semantics is that it does not validate STRENGTHENING, but at the same time one of its major weaknesses is that it does not validate SIMPLIFICATION either.

As we have seen, this dilemma is resolved if a minimal change semantics of conditionals is incorporated into the general framework proposed here, which does not equate semantic meaning with informative content, but takes inquisitive content into account as well. The inquisitive treatment of disjunction and implication validates SIMPLIFICATION, while the stronger STRENGTHENING remains invalid.

5 Types of proposals

5.1 Suppositionality, informativeness, and inquisitiveness

Definition 17 (Suppositional, informative, and inquisitive sentences)**.**

- φ is suppositional iff $\mathsf{sup}(\varphi) \neq \omega$;
- φ is informative iff $\mathsf{info}(\varphi) \neq \omega$;
- φ is inquisitive iff $\mathsf{info}(\varphi) \notin \lceil\varphi\rceil$;

[17] Although see Michael Franke's dissertation, page 170, who cites the following example from McKay and van Inwagen as an argument against SIMPLIFICATION:

(i) If John had taken an apple or a pear, he would have taken an apple.
$\not\leadsto$ If John had taken a pear, he would have taken an apple.

What to say about this case?

Michael also discusses an argument by Warmbrot to the effect that SIMPLIFICATION should be invalid. The argument is that, if negation, conjunction, and disjunction are interpreted classically, then SIMPLIFICATION actually entails STRENGTHENING. This argument may not be so relevant here, as negation, conjunction, and disjunction are *not* interpreted classically.

- φ is attentive iff there is a possibility for φ that is strictly included in a maximal possibility fot φ;
- φ is counter-inquisitive iff $\mathsf{contra}(\varphi) \notin \lfloor\varphi\rfloor$.

5.2 Questions, assertions, and hybrids

Questions, assertions, and hybrids are defined as usual.

Definition 18 (Questions, assertions, and conjectures)**.**

- φ is a question iff it is not informative and not attentive;
- φ is a assertion iff it is not inquisitive and not attentive;
- φ is a conjecture iff it is not inquisitive and not informative;

Note that within each of these categories we can now make a distinction between suppositional and non-suppositional instances. That is, we can distinguish suppositional questions from non-suppositional questions, suppositional assertions from non-suppositional assertions, and suppositional conjectures from non-suppositional conjectures.

6 Assessing a proposal

Let us now consider the assessment of a given proposal by a responder, relative to his information state. Throughout, we will refer to the responder as R, and to his information state as ρ. We will use $[\varphi]$ to denote the proposal expressed by φ.

6.1 Licity, acceptability, and objectionability

Definition 19 (Licit and illicit proposals)**.**

- $[\varphi]$ is *licit* in ρ iff ρ is consistent with the supposition of φ: $\rho \cap \mathsf{sup}(\varphi) \neq \emptyset$
- $[\varphi]$ is *illicit* in ρ iff it is not licit in ρ.

Definition 20 (Acceptable and unacceptable proposals)**.**

- $[\varphi]$ is *acceptable* in ρ iff

– $[\varphi]$ is licit in ρ, and
– ρ does not support the counter-information for φ: $\rho \not\subseteq \mathsf{contra}(\varphi)$

- $[\varphi]$ is *unacceptable* in ρ iff
  – $[\varphi]$ is licit in ρ
  – ρ supports the counter-information for φ: $\rho \subseteq \mathsf{contra}(\varphi)$

Fact 21 (Licity and acceptability)**.**

- If $[\varphi]$ is illicit in ρ, then it is neither acceptable nor unacceptable in ρ.
- If $[\varphi]$ is licit in ρ, then it is either acceptable or unacceptable in ρ.

Definition 22 (Objectionable and unobjectionable proposals)**.**

- $[\varphi]$ is *objectionable* in ρ iff
  – $[\varphi]$ is licit in ρ
  – There is a sub-state $\varrho \subseteq \rho$ such that $[\varphi]$ is unacceptable in ϱ
- $[\varphi]$ is *unobjectionable* in ρ iff
  – $[\varphi]$ is licit in ρ
  – There is no sub-state $\varrho \subseteq \rho$ such that $[\varphi]$ is unacceptable in ϱ

Fact 23 (Licity and objectionability)**.**

- If $[\varphi]$ is illicit in ρ, then it is neither objectionable nor unobjectionable in ρ.
- If $[\varphi]$ is licit in ρ, then it is either objectionable or unobjectionable in ρ.

Fact 24 (Acceptability and objectionability)**.**

- If $[\varphi]$ is unacceptable in ρ, then it is objectionable in ρ.
- If $[\varphi]$ is unobjectionable in ρ, then it is acceptable in ρ.
- It is possible for $[\varphi]$ to be both acceptable and objectionable in ρ.

Definition 25 (Contingent proposals)**.**

- $[\varphi]$ is *contingent* in ρ iff it is both acceptable and objectionable in ρ.

Example 26 (A contingent proposal)**.**
The proposal expressed by the atomic sentence p is contingent in the ‘ignorant’ information state ω. First of all, p does not have any (non-trivial) supposition, so $[p]$ is licit in ω. Furthermore, $\mathsf{contra}(p) = |{\sim}p|$, which is not supported by ω. So $[p]$ is acceptable in ω. On the other hand, there are sub-states of ω where $[p]$ is unacceptable. Take, for instance, the state $|{\sim}p|$. In this state, $[p]$ is still licit. But, of course $|{\sim}p|$ supports $\mathsf{contra}(p)$, which means that $[p]$ is unacceptable in $|{\sim}p|$, and therefore that it is objectionable in ω. Conclusion: $[p]$ is both acceptable and objectionable in ω, and therefore contingent in ω.

The resulting categorization is depicted below:

<table>
<tr><td></td><td>Illicit</td><td></td></tr>
<tr><td rowspan="2">Objectionable</td><td>Unacceptable</td><td></td></tr>
<tr><td>Contingent</td><td rowspan="2">Acceptable</td></tr>
<tr><td></td><td>Unobjectionable</td></tr>
</table>

Fact 27 (Objectionability, contingency, and acceptability)**.**

- $[\varphi]$ is objectionable in ρ iff it is unacceptable or contingent in ρ.
- $[\varphi]$ is acceptable in ρ iff it is contingent or unobjectionable in ρ.

Notice that licity was defined in terms of $\mathsf{sup}(\varphi)$, and that acceptability and objectionability were defined in terms of $\mathsf{sup}(\varphi)$ and $\mathsf{contra}(\varphi)$. In particular, none of these notions make any reference to $\mathsf{info}(\varphi)$. In a classical setting, $\mathsf{info}(\varphi)$ is everything that the semantics gives us. Thus, we clearly see a payoff here of the enriched semantics.

6.2 Support and unobjectionability

Definition 28 (Supported proposals)**.**

- $[\varphi]$ is *supported* by ρ iff
  - $[\varphi]$ is licit in ρ, and
  - $\rho \subseteq \mathsf{info}(\varphi)$

Fact 29 (Support and unobjectionability)**.**
$[\varphi]$ is supported by ρ iff it is unobjectionable in ρ.

Proof. Suppose that $[\varphi]$ is supported in ρ. Then $\rho \cap \mathsf{sup}(\varphi) \neq \emptyset$ and $\rho \subseteq \mathsf{info}(\varphi)$. To show: there is no sub-state ϱ of ρ such that $\varrho \cap \mathsf{sup}(\varphi) \neq \emptyset$ and $\varrho \subseteq \mathsf{contra}(\varphi)$. Towards a contradiction, suppose that there is such a state ϱ. Then we have that $\varrho \subseteq \mathsf{info}(\varphi)$ and $\varrho \subseteq \mathsf{contra}(\varphi)$. This means that $\varrho \subseteq \mathsf{info}(\varphi) \cap \mathsf{contra}(\varphi)$. But $\mathsf{info}(\varphi) \cap \mathsf{contra}(\varphi)$ is the complement of $\mathsf{sup}(\varphi)$. So $\varrho \cap \mathsf{sup}(\varphi) = \emptyset$, which contradicts one of our assumptions about ϱ. Conclusion: if $[\varphi]$ is supported in ρ then it must be unobjectionable in ρ.

Conversely, suppose that $[\varphi]$ is unobjectionable in ρ. Then $\rho \cap \mathsf{sup}(\varphi) \neq \emptyset$ and there is no sub-state ϱ of ρ such that $\varrho \cap \mathsf{sup}(\varphi) \neq \emptyset$ and $\varrho \subseteq \mathsf{contra}(\varphi)$. To show: $\rho \subseteq \mathsf{info}(\varphi)$. Towards a contradiction, suppose that $\rho \nsubseteq \mathsf{info}(\varphi)$. This means that there must be a sub-state ϱ of ρ such that $\varrho \cap \mathsf{info}(\varphi) = \emptyset$. In other words, $\varrho \subseteq \overline{\mathsf{info}(\varphi)}$. But $\overline{\mathsf{info}(\varphi)} = \mathsf{sup}(\varphi) \cap \mathsf{contra}(\varphi)$. So ϱ is consistent with $\mathsf{sup}(\varphi)$ and is contained in $\mathsf{contra}(\varphi)$, which is in contradiction with our assumptions. Conclusion: if $[\varphi]$ is unobjectionable in ρ, then it must be supported by ρ. □

6.3 Resolvability and counter-resolvability

Definition 30 (Resolvability)**.**

- $[\varphi]$ is resolvable in ρ iff there is at least one possibility $\alpha \in \lceil\varphi\rceil$ such that either $\rho \subseteq \alpha$ or $\rho \cap \alpha = \emptyset$;
- $[\varphi]$ is completely resolvable in ρ iff every possibility $\alpha \in \lceil\varphi\rceil$ is such that either $\rho \subseteq \alpha$ or $\rho \cap \alpha = \emptyset$.

Definition 31 (Counter-resolvability)**.**

- $[\varphi]$ is counter-resolvable in ρ iff there is at least one counter-possibility $\alpha \in \lfloor\varphi\rfloor$ such that either $\rho \subseteq \alpha$ or $\rho \cap \alpha = \emptyset$;
- $[\varphi]$ is completely counter-resolvable in ρ iff every counter-possibility $\alpha \in \lfloor\varphi\rfloor$ is such that either $\rho \subseteq \alpha$ or $\rho \cap \alpha = \emptyset$.

Notice that resolvability and counter-resolvability are defined in terms of $\lceil\varphi\rceil$ and $\lfloor\varphi\rfloor$, respectively. Again, we see a payoff here of our enriched semantics. A classical semantics only delivers $\mathsf{info}(\varphi)$, which is much too coarse-grained to serve as a basis for a characterization of resolvability and counter-resolvability.

7 Types of responses

NEW IN APRIL:

Let us consider two cases separately: responses to contingent proposals, and responses to non-contingent proposals, starting with the latter.

7.1 Responding to non-contingent proposals

Non-contingent proposals are either illicit, or unacceptable, or unobjectionable (recall that a proposal is unobjectionable if and only if it is supported). These three different statuses call for three different types of responses.

If $[\varphi]$ is *illicit* in ρ then the most suitable way to respond is to *refuse* the given proposal.

Definition 32 (Refusal). ψ refuses $[\varphi]$ iff $\mathsf{info}(\psi) \cap \mathsf{sup}(\varphi) = \emptyset$

If $[\varphi]$ is *unacceptable* in ρ then the most suitable way to respond is to *reject* the given proposal.

Definition 33 (Rejection). ψ rejects $[\varphi]$ iff

- $\mathsf{info}(\psi) \cap \mathsf{sup}(\varphi) \neq \emptyset$, and
- $\mathsf{info}(\psi) \subseteq \mathsf{contra}(\varphi)$

If $[\varphi]$ is *unobjectionable* in ρ then the most suitable way to respond is to *endorse* the given proposal.

Definition 34 (Endorsement). ψ endorses $[\varphi]$ iff

- $\mathsf{info}(\psi) \cap \mathsf{sup}(\varphi) \neq \emptyset$, and
- $\mathsf{info}(\psi) \subseteq \mathsf{info}(\varphi)$

7.2 Logically relatedness, compliance and counter-compliance

Orthogonal to the distinction between refusal, rejection, and endorsement, we could make another general distinction between responses that are *logically related* to the given proposal and ones that are not. We define logical relatedness in terms of the question behind the given proposal:

Definition 35 (Logical relatedness)**.**
ψ is logically related to φ iff every possibility for ψ can be obtained from a set of possibilities for $?\varphi$ by applying union, intersection, and complementation.

With this notion of logical relatedness in hand, we can define suitable notions of compliance and counter-compliance.

Definition 36 (Compliance)**.**
ψ is a compliant response to φ iff it is logically related to φ and endorses $[\varphi]$.

Definition 37 (Counter-compliance)**.**
ψ is a counter-compliant response to φ iff it is logically related to φ and rejects $[\varphi]$.

Not all logically related responses are either compliant or counter-compliant. There are also logically related responses that refuse the given proposal (e.g., $\sim p$ in response to $p \to q$) and ones that neither refuse nor reject nor endorse the given proposal (e.g., p in response to $p \to q$ or in response to $p \wedge q$).

To summarize, then, we have:

$$\text{compliant} = \text{logically related} \cap \text{endorsing}$$

$$\text{counter-compliant} = \text{logically related} \cap \text{rejecting}$$

$$\text{logically related} \neq \text{compliant} \cup \text{counter-compliant}$$

We are now also ready to give a refined definition of *basic* compliant and counter-compliant responses:

Definition 38 (Basic responses)**.**
ψ is a basic response to φ iff $\lceil\psi\rceil$ consists of a single possibility α, and $\alpha \in \lceil ?\varphi \rceil$.

Definition 39 (Basic compliance)**.**
ψ is a basic compliant response to φ iff it is a basic response to ψ that endorses $[\varphi]$.

Definition 40 (Basic counter-compliance)**.**
ψ is a basic counter-compliant response to φ iff it is a basic response to φ that rejects $[\varphi]$.

The difference with our preliminary definitions in section 2 is that basic responses to φ which *refuse* $[\varphi]$ no longer count as basic compliant or counter-compliant responses.

Fact 41 (Compliance subsumes basic compliance)**.**
Any basic compliant response to φ is a compliant response to φ.

Fact 42 (Counter-compliance subsumes basic counter-compliance)**.**
Any basic counter-compliant response to φ is a counter-compliant response to φ.

7.3 Responding to contingent proposals

- If $[\varphi]$ is *contingent* in ρ then there are two ways to react to it. One option is to *accept* the proposal, which amounts to updating ρ with $\mathsf{info}(\varphi)$. Acceptance is the *default* respnse: it can be signalled by nodding or saying something like *okay* or *right*. But even in the absence of any explicit signal, other participants assume that the proposal was accepted.

- The other way to react if $[\varphi]$ is contingent in ρ is to *object* to it. This amounts to indicating that, even though $[\varphi]$ is not unacceptable in ρ itself, there is a sub-state $\varrho \subseteq \rho$ in which $[\varphi]$ *is* unacceptable. This means that, as more information is acquired, $[\varphi]$ may become unacceptable.

- Whether a responder will accept $[\varphi]$ or object to it will depend mostly on whether he *trusts* the initiator.

Summing up, we have the following typology:[18]

(92) **Typology of responses**
  a. Response-types for non-contingent proposals:
    (i) Refusal
    (ii) Rejection
    (iii) Endorsement
  b. Response-types for contingent proposals:
    (i) Acceptance
    (ii) Objection

[18] We should add *partial endorsement* and *partial rejection*. The former would have to cover p as a response to $p \wedge q$, the latter would have to cover $\sim p$ as a response to $p \vee q$.

In natural language, there are at least two ways in which objections can be articulated. This is perhaps best illustrated by means of an example.

Example 43 (Two ways to articulate objections in natural language)**.**
Suppose that someone says:

(93) Pete will play the piano tonight. p

Suppose furthermore that the responder knows that Pete will only play the piano if Sue will sing, and has no information yet as to whether or not Sue will sing. In such a situation, the responder may happily accept the given proposal and update with the information that Pete will play the piano (and that, apparently, Sue will sing as well). But he may also object. There are at least two typical ways in which such an objection may be articulated:

(94) Well, if Sue won't sing then Pete won't play the piano. $\sim q \to \sim p$

(95) Well, it might be that Pete won't play tonight. $\Diamond \sim p$

In general objections can be charaterized as follows:

Definition 44 (Objections)**.**
ψ is an objection to φ iff

- ψ does not reject φ, i.e., $\mathsf{info}(\psi) \not\subseteq \mathsf{contra}(\varphi)$, and
- $\mathsf{sup}(\psi) \cap \mathsf{info}(\psi) \subseteq \mathsf{contra}(\varphi)$.

According to this characterization objections are generally suppositional, they do not signal that the given proposal is altogether unacceptable, but rather that it is unacceptable under certain suppositions.

Whether this characterization applies to the objection in (95) depends, of course, on the semantics that we assign to sentences like $\Diamond \sim p$, and in particular on the supposition that such sentences express.

Might. The most natural clauses for $\Diamond\varphi$ would presumably be the following:

- $\lceil \Diamond\varphi \rceil = \lceil \top \vee \varphi \rceil$
- $\lfloor \Diamond\varphi \rfloor = \lfloor \varphi \rfloor$

This has interesting consequences:

- $\lfloor ?\Diamond\varphi \rfloor = \lfloor \Diamond\varphi \vee {\sim}\Diamond\varphi \rfloor$
$$= \{\alpha \cap \beta \mid \alpha \in \lfloor \Diamond\varphi \rfloor \text{ and } \beta \in \lfloor {\sim}\Diamond\varphi \rfloor\}$$
$$= \{\alpha \cap \beta \mid \alpha \in \lfloor \varphi \rfloor \text{ and } \beta \in \lceil \Diamond\varphi \rceil\}$$
$$= \{\alpha \cap \beta \mid \alpha \in \lfloor \varphi \rfloor \text{ and } \beta \in \lceil \top \vee \varphi \rceil\}$$
- This means that $\bigcup \lfloor ?\Diamond\varphi \rfloor = \bigcup \lfloor \varphi \rfloor$
- In other words: $\mathsf{sup}(\Diamond\varphi) = \overline{\mathsf{contra}(\varphi)}$

Thus, countering $\Diamond\varphi$ always amounts to canceling its supposition. That is, $\Diamond\varphi$ can never be rejected, it can only be refused.

Notice that $\mathsf{sup}(\Diamond{\sim}p) = |{\sim}p|$. So the above characterization of objections correctly recognizes $\Diamond{\sim}p$ as an objection to $[p]$.

Negating *might*. We lose the result that ${\sim}\Diamond\varphi$ is always a contradiction, which we used in the conservative setting to explain that standard negation cannot scope over *might* in English. Can this be explained in some other way?

Yes, and indeed very straightforwardly: one of the counter-possibilities for ${\sim}\Diamond\varphi$ will always be ω, which means that the information state of someone who utters ${\sim}\Diamond\varphi$ will always support a counter-possibility for this sentence. In other words, someone who utters ${\sim}\Diamond\varphi$ always knows exactly how his own proposal can be rejected. In this sense, ${\sim}\Diamond\varphi$ is 'unassertable'.

Questioning *might*. A similar story has to be told for $?\Diamond\varphi$. In the non-radical setting, we have that $?\Diamond\varphi \equiv \Diamond\varphi$. This we took to be the trigger for re-interpretation of *might* in the scope of a question. However, in the radical setting, it is no longer generally the case that $?\Diamond\varphi \equiv \Diamond\varphi$.

One option would be to start thinking of ? as the *purely inquisitive* closure operator instead of the *non-informative* closure operator. Then we would get that $\lceil ?\Diamond\varphi \rceil = \{\omega\}$ for every φ, both in the radical and in the non-radical and this would be enough to trigger re-interpretation.

Epistemic contradictions. There are at least two ways in which we could try to account for epistemic contradictions.

- We could account for the unassertability of sentences like

(96) $\Diamond p \wedge {\sim}p$

in much the same way as we accounted for the unassertability of $?\Diamond p$. Namely, in order to respect informative sincerity, the information state of a speaker who utters (96) must be contained in $\mathsf{info}(\Diamond p \wedge {\sim}p)$, which amounts to $|{\sim}p|$. However, $|{\sim}p|$ is also a counter-possibility for (96). So a speaker can only utter the sentence sincerely if he already knows exactly how it can be rejected.

- However, this approach does not generalize to cases like:

  (97) $\Diamond(p \vee q) \wedge {\sim}p$

  The informative content of this sentence is $|{\sim}p|$, while its only counter-possibility is assumed to be $|{\sim}p \wedge {\sim}q|$. So someone who utters the sentence and supports its informative content is not necessarily in a position to reject it.

- The alternative account is to take the *empty possibility* as a sign of inconsistency, even if it appears among other possibilities. After all, if $\lceil\varphi\rceil$ contains the empty possibility, this can only be because φ draws attention to a certain possibility and at the same time it provides enough information to completely exclude that possibility. Intuitively, this is exactly what happens with sentences like (96) and (97). And indeed, in both cases we find that $\lceil\varphi\rceil$ includes the empty possibility.

  Given attentive sincerity, whenever $\lceil\varphi\rceil$ includes the empty possibility, φ is unassertable.

- Notice that Yalcin's account of (96) does not generalize to (97).

- The second account does not make reference to counter-possibilities. It works in the non-radical setting just as well.

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